THE PROCEEDINGS OF
THE PHYSICAL SOCIETY
Section B

VOL. 64, PART 12 1 December 1951 No. 384 B

Ultrasonic Relaxation Processes in Pure Liquids

BY J. H. ANDREAE AND JOHN LAMB
Electrical Engineering Department, Imperial College, London

Communicated by Willis Jackson; MS. received 27th June 1951

SUMMARY. The assumptions made by Hall and Herzfeld in previous theories of ultrasonic absorption are interpreted on the basis of a new and more general analysis. The formulae given by them are discussed, and it is suggested that the conditions implicit in Herzfeld's formula should be modified. By this modification a new relation is derived, which provides a satisfactory explanation of the observed relaxation effects in those liquids which have been studied.

A method of 'curve fitting' is employed to compare experimental results with theoretical requirements. Finally, it is concluded that the ultrasonic behaviour of acetic acid and propionic acid is governed by the excitation of bending vibrations of carbon–hydrogen bonds. Measurements in carbon disulphide indicate that in this case also a bending vibration is responsible. Some incomplete data for toluene suggest that here a stretching vibration of the carbon–hydrogen bonds in the methyl group is involved.

§1. INTRODUCTION

A CONSIDERABLE amount of experimental data has been published on the absorption and velocity of ultrasonic waves in pure liquids. In most cases, however, measurements have been taken at single frequencies or temperatures giving little systematic information. A satisfactory correlation of experimental results with existing theory requires a knowledge of the frequency and temperature dependence of the absorption and velocity over a complete region of dispersion: only in the case of acetic and propionic acids (Lamb and Pinkerton 1949, Lamb and Huddart 1950) has this been provided as yet.

It was shown that the measured dispersion in acetic and propionic acids can be interpreted on the basis of a relaxation of the transfer of energy between two postulated states; but in calculating the difference of free energy between these two states a large entropy factor had to be invoked, which is difficult to explain. It is well known that the carboxylic acids exhibit association as double molecules and so it was natural to suspect (Spakowski 1938) that the equilibrium between the monomeric and dimeric states might be responsible for the relaxation. An examination of the results on this assumption does not, however, appear to account for the entropy and, further, there is evidence of dispersions in ethyl and methyl acetates (Biquard 1936, Beyer and Smith 1946) and in ethyl formate (Liebermann 1949), where double molecules do not occur.

Hall (1948) has calculated the effect of the relaxation of an isothermal change in the structure of water, but it is questionable whether the stringent isothermal restriction is applicable in the general case. It seems more likely that a process allowing for a change of specific heat together with a change of volume would be responsible for the observed relaxations. The following general theoretical treatment was prompted by recent analyses given by Kneser (1950) and Hall (1948).

§ 2. THEORETICAL ANALYSIS

2.1. *Derivation of the General Equation*

It will be assumed that the molecules of the liquid can exist in either of two states, designated 1 and 2, and that the transition of molecules between these two states can be described by the first order reaction equation:

$$\frac{dn_2}{dt}=k_{21}n_1\left[K-\frac{n_2}{n_1}\right]. \qquad \ldots\ldots(1)$$

n_1 and n_2 are the numbers of molecules per mole in states 1 and 2, respectively; k_{12} and k_{21} are the appropriate rate constants and $K(=k_{12}/k_{21})$ is the equilibrium constant. The total number of molecules per mole, $N(=n_1+n_2)$, is unaffected by the transitions. In the presence of a sound* wave of frequency $\omega/2\pi$, equation (1) becomes $j\omega\, dn_2=k_{21}n_1\{dK-d(n_2/n_1)\}$, from which is derived the relation

$$dn_2=\frac{NK}{(1+K)^2}\frac{d(\ln K)}{1+j\omega\tau}, \qquad \ldots\ldots(2)$$

where $\tau=1/(k_{12}+k_{21})$.

Now the effective adiabatic compressibility β at frequency $\omega/2\pi$ is governed by the above reaction and can be separated into two parts as follows:

$$\beta=-\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S=-\frac{1}{V}\left(\frac{\partial V}{\partial n_2}\right)_x\left(\frac{\partial n_2}{\partial P}\right)_S-\frac{1}{V}\left(\frac{\partial V}{\partial x}\right)_{n_2}\left(\frac{\partial x}{\partial P}\right)_S$$

where P, V, S have their usual thermodynamic significance and the quantity x is defined in terms of that contribution to the compressibility β_∞ which is not affected by transitions between the two states: thus

$$\beta_\infty=-\frac{1}{V}\left(\frac{\partial V}{\partial x}\right)_{n_2}\left(\frac{\partial x}{\partial P}\right)_S.$$

$$\beta_r=\beta-\beta_\infty=-\frac{1}{V}\left(\frac{\partial V}{\partial n_2}\right)_x\left(\frac{\partial n_2}{\partial P}\right)_S,^\dagger \qquad \ldots\ldots(3)$$

is called the 'relaxing component of the compressibility', and by virtue of equation (2) will be shown later to have the form

$$\beta_r=\beta_1/(1+j\omega\tau)=(\beta_0-\beta_\infty)/(1+j\omega\tau), \qquad \ldots\ldots(4)$$

where β_0 is the static adiabatic compressibility.

A complex velocity of the sound wave V_c is conveniently defined as

$$\frac{1}{V_c}=\frac{1}{c}-j\frac{\alpha}{\omega}=\frac{1}{c}\left(1-j\frac{\alpha c}{\omega}\right), \qquad \ldots\ldots(5)$$

where c is the phase velocity and α the (pressure) attenuation coefficient. The absorption per wavelength μ is given by

$$\mu=2\pi\alpha c/\omega=-\pi\,\mathrm{Im}\,(c^2/V_c^{\,2}), \qquad \ldots\ldots(6)$$

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* The word 'sound' is frequently used in the wider sense embracing both audible and ultrasonic waves.

† Kneser (1950) appears to make the error of putting

$$\beta_0-\beta_\infty=-\frac{1}{V}\left(\frac{\partial V}{\partial n_2}\right)_S\left(\frac{\partial n_2}{\partial P}\right)_{S_{\omega=0}} \text{ instead of } \beta_0-\beta_\infty=-\frac{1}{V}\left(\frac{\partial V}{\partial n_2}\right)_x\left(\frac{\partial n_2}{\partial P}\right)_{S_{\omega=0}}$$

provided $(\alpha c/\omega)^2 \ll 1$. It follows from the above, if ρ is the density, that $1/V_c^2 = \rho\beta = \rho\{\beta_\infty + \beta_i/(1+j\omega\tau)\}$ whence $\mu = \{\omega\bar{\tau}/(1+\omega^2\bar{\tau}^2)\}\pi\beta_i/(\beta_0\beta_\infty)^{1/2}$, and $\bar{\tau} = \tau(\beta_\infty/\beta_0)^{1/2}$. When $\omega\bar{\tau} = 1$, μ has a maximum value

$$\mu_m = \tfrac{1}{2}\pi\beta_i/(\beta_0\beta_\infty)^{1/2} \simeq \tfrac{1}{2}\pi\beta_i/\beta_0 \qquad \ldots\ldots(7)$$

if $\beta_i \ll \beta_0$. The associated dispersion in phase velocity is given by

$$(c_\infty - c_0)/c_0 = (\beta_0/\beta_\infty)^{1/2} - 1 \simeq \mu_m/\pi$$

where $c_0^2 = 1/\rho\beta_0$, $c_\infty^2 = 1/\rho\beta_\infty$.

It is now required to derive an expression for β_i/β_0 to account for the measured values of μ_m. The factor $(\partial n_2/\partial P)_S$ appearing in β_r (equation (3)) is obtained from equation (2) as

$$\frac{1}{N}\left(\frac{\partial n_2}{\partial P}\right)_S = \left[\left(\frac{\partial(\ln K)}{\partial P}\right)_T + \left(\frac{\partial(\ln K)}{\partial T}\right)_P\left(\frac{\partial T}{\partial P}\right)_S\right]\frac{K}{(1+K)^2}\frac{1}{1+j\omega\tau}.$$

Using the Van't Hoff equation $[\partial(\ln K)/\partial T]_P = \Delta H/\boldsymbol{R}T^2$ and its isothermal analogue $[\partial(\ln K)/\partial P]_T = -\Delta V/\boldsymbol{R}T$ the previous relation becomes

$$\frac{1}{N}\left(\frac{\partial n_2}{\partial P}\right)_S = \left[-\frac{\Delta V}{\boldsymbol{R}T} + \frac{\Delta H}{\boldsymbol{R}T^2}\left(\frac{\partial T}{\partial P}\right)_S\right]\frac{\exp(-\Delta H/\boldsymbol{R}T)}{\{1+\exp(-\Delta H/\boldsymbol{R}T^2)\}}\frac{1}{1+j\omega\tau} \qquad \ldots\ldots(8)$$

where $$K = \exp(-\Delta H/\boldsymbol{R}T)$$

and $$\Delta H = \Delta E + P^*\Delta V = C_v\Delta T + (P_i + P)\Delta V \qquad \ldots\ldots(9)$$

is the heat of reaction per mole for the transition from state 1 to state 2; ΔE is the change of internal energy, at constant volume, and ΔV is the isothermal volume change accompanying the reaction; the internal pressure P_i is defined by $P_i + P = P^* = T(\partial S/\partial V)_T$.

The heat of reaction for the transition can be expressed in terms of an entropy change dS where

$$T\,dS = T\left(\frac{\partial S}{\partial V}\right)_P dV + T\left(\frac{\partial S}{\partial P}\right)_V dP = \mathrm{T}\left(\frac{\partial P}{\partial T}\right)_S dV - T\left(\frac{\partial V}{\partial T}\right)_S dP = \Delta H\frac{dn_2}{N}.$$

Hence $$N\left(\frac{\partial V}{\partial n_2}\right)_x = \frac{\Delta H}{T(\partial P/\partial T)_S} + \left(\frac{\partial V}{\partial P}\right)_S N\left(\frac{\partial P}{\partial n_2}\right)_x. \qquad \ldots\ldots(10)$$

Putting $N(\partial P/\partial n_2)_x = \Delta P$, and combining equations (3), (8) and (10),

$$\beta_i = \left[\frac{\Delta H}{VT(\partial P/\partial T)_S} - \beta_0\Delta P\right]\left[\frac{\Delta V}{\boldsymbol{R}T} - \frac{\Delta H}{\boldsymbol{R}T^2}\left(\frac{\partial T}{\partial P}\right)_S\right]\frac{\exp(-\Delta H/\boldsymbol{R}T)}{\{1+\exp(-\Delta H/\boldsymbol{R}T)\}^2}.$$

Finally, rearrangement of this equation gives:

$$\frac{\beta_i}{\beta_0} = \left[(\gamma-1) - \gamma\beta_0 P^*\frac{V\Delta P}{\Delta H}\right]\left[\frac{\gamma}{(\gamma-1)}\frac{P^*\Delta V}{\Delta H} - 1\right]\frac{C_i}{C_p} \simeq \frac{2\mu_m}{\pi} \qquad \ldots\ldots(11)$$

where $$C_i = \boldsymbol{R}\left(\frac{\Delta H}{\boldsymbol{R}T}\right)^2\frac{\exp(-\Delta H/\boldsymbol{R}T)}{\{1-\exp(-\Delta H/\boldsymbol{R}T)\}^2} \qquad \ldots\ldots(12)$$

is the relaxing component of the specific heat and $\gamma\ (=C_p/C_v)$ is the ratio of the static specific heats.

The formula (12) for C_i was obtained on the assumption that there is an equilibrium between two states. In certain cases, where the heat of reaction may result from the excitation of vibrational energy levels of the molecules, there is the possibility that the higher energy states may become important. The maximum contribution which the energy levels of a particular vibrational mode can make to the relaxing specific heat C_i, is given by the Einstein relation

$$C_i = \boldsymbol{R}\left(\frac{\Delta E}{\boldsymbol{R}T}\right)^2 \frac{\exp(-\Delta E/\boldsymbol{R}T)}{\{1-\exp(-\Delta E/\boldsymbol{R}T)\}^2} \qquad \ldots\ldots(13)$$

where $\Delta E = \boldsymbol{h}\nu$ and ν is the fundamental frequency for the mode in question. In the case of a degenerate vibration the Einstein specific heat has to be multiplied by the degeneracy n; when only the lowest two states are considered the reaction equation (1) applies with $K = n\exp(-\Delta E/\boldsymbol{R}T)$, giving

$$C_i = nR\left(\frac{\Delta E}{\boldsymbol{R}T}\right)^2 \frac{\exp(-\Delta E/\boldsymbol{R}T)}{\{1+n\exp(-\Delta E/\boldsymbol{R}T)\}^2} \qquad \ldots\ldots(14)$$

for the relaxing specific heat.

In addition to the above there is the possibility that transitions between two postulated states will be accompanied by a change in the number of molecules present. For example, the dissociation of the double molecules of acetic and propionic acids into single molecules would be described by the reaction equation $dn_2/dt = k_{21}(n_1+n_2)n_1\{K - n_2^2/(n_1+n_2)n_1\}$ where the equilibrium constant $K = k_{12}/k_{21}$ as before; n_1 and n_2 are the numbers of double and single molecules, respectively, and $N(=2n_1+n_2)$ the *total* number of single molecules including those forming double molecules is constant. It can be shown that the relation corresponding to equation (2) for this case is

$$dn_2 = \frac{1}{4}\frac{NK^{1/2}}{(1+K/4)^{3/2}}\frac{d(\ln K)}{1+j\omega\tau}$$

and the contribution to the relaxing specific heat is

$$C_i = \boldsymbol{R}\left(\frac{\Delta H}{\boldsymbol{R}T}\right)^2 \frac{\exp(-\Delta H/\boldsymbol{R}T)}{[1+\{\frac{1}{2}\exp(-\Delta H/\boldsymbol{R}T)\}^2]^{3/2}} \qquad \ldots\ldots(15)$$

where ΔH is *half* the heat of dissociation per mole.

2.2. *Special Cases*

(*a*) *Hall's isothermal expression for* $\Delta E = 0$. The formula for an isothermal process follows from the general equation (11) by putting $\Delta E = 0$ and $-V^{-1}(\partial V/\partial P)_T = -V^{-1}(\Delta V/\Delta P)$; this gives

$$\left[\frac{\beta_i}{\beta_0}\right]_{\text{isothermal}} = \frac{\beta_i'}{\beta_0'} = \frac{\gamma}{\gamma-1}\frac{C_i}{C_p} = \frac{\beta_0'}{\epsilon^2 VT}\boldsymbol{R}\left(\frac{\Delta H}{\boldsymbol{R}T}\right)^2 \frac{\exp(-\Delta H/\boldsymbol{R}T)}{\{1+\exp(-\Delta H/\boldsymbol{R}T)\}^2} \qquad \ldots\ldots(16)$$

where ϵ is the thermal expansion coefficient, and finally

$$\beta_i' = \frac{V}{\boldsymbol{R}T}\left(\frac{\Delta V}{V}\right)^2 \frac{1}{2\{1+\cosh(\Delta H/\boldsymbol{R}T)\}},$$

which is identical with that given by Hall (1948).

(*b*) *Herzfeld's expression for* $\Delta V = 0$. By assuming that the isothermal compressibility is unaffected by the relaxing process in the liquid, Herzfeld (1941) has derived a formula equivalent to

$$\beta_i/\beta_0 = (\gamma-1)C_i/C_p. \qquad \ldots\ldots(17)$$

This relation is obtained from the general equation (11) by putting $\Delta V = 0$ and $2T(\partial P/\partial T)_V = T\Delta P/\Delta T$.

(*c*) ***Alternative expression for*** $\Delta V=0$. The assumption $2T(\partial P/\partial T)_V=T\Delta P/\Delta T$ upon which Herzfeld's formula (17) appears to be based seems somewhat arbitrary. By analogy with the condition $-V^{-1}(\partial V/\partial P)_T=-V^{-1}\Delta V/\Delta P$ used above to give the isothermal relation, it is suggested that in the case where $\Delta V=0$, the conditions $T\Delta P/\Delta T=T(\partial P/\partial T)_V$ or $T\Delta P/\Delta T=T(\partial P/\partial T)_S$ would be more appropriate.* The former does not permit a contribution to the compressibility (i.e. $\beta_i=0$), but the latter gives

$$\beta_i/\beta_0=C_i/C_p. \qquad \ldots\ldots(18)$$

In the following sections it will be shown that this equation leads to the best agreement with experiment.

In order to facilitate a comparison of the equations derived in this section with experimental results, the chief points of the preceding analysis are summarized in Tables 1 and 2.

SUMMARY OF THEORETICAL ANALYSIS

General Equation

$$\frac{2\mu_m}{\pi}=\frac{\beta_i}{(\beta_0\beta_\infty)^{1/2}}\simeq\frac{\beta_i}{\beta_0}=\left[(\gamma-1)-\gamma\beta_0P^*\frac{V\Delta P}{\Delta H}\right]\left[\frac{\gamma}{\gamma-1}\frac{P^*\Delta V}{\Delta H}-1\right]\frac{C_i}{C_p}$$

$$\Delta H=\Delta E+P^*\Delta V$$

Table 1. Special Cases

	Assumptions		Resulting equation	
(*a*)	$\Delta E=0$ $-\frac{1}{V}\frac{\Delta V}{\Delta P}=-\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$	Isothermal relaxation	$\frac{\beta_i}{\beta_0}=\frac{\gamma}{\gamma-1}\frac{C_i}{C_p}$	(16)
(*b*)	$\Delta V=0$ $T\frac{\Delta P}{\Delta T}=2T\left(\frac{\partial P}{\partial T}\right)_V$	Isothermal compressibility unaffected by relaxing process	$\frac{\beta_i}{\beta_0}=(\gamma-1)\frac{C_i}{C_p}$	(17)
(*c*)	$\Delta V=0$			
	(i) $T\frac{\Delta P}{\Delta T}=T\left(\frac{\partial P}{\partial T}\right)_V$		$\beta_i=0$	
	(ii) $T\frac{\Delta P}{\Delta T}=T\left(\frac{\partial P}{\partial T}\right)_S$		$\frac{\beta_i}{\beta_0}=\frac{C_i}{C_p}$	(18)

Table 2. Relaxing Component of Specific Heat

	Assumptions	Associated specific heat	
(1)	Two-state reaction: general case. ΔH = heat of reaction	$C_i=\boldsymbol{R}\left(\frac{\Delta H}{\boldsymbol{R}T}\right)^2\frac{\exp(-\Delta H/\boldsymbol{R}T)}{\{1+\exp(-\Delta H/\boldsymbol{R}T)\}^2}$	(12)
(2)	Two-state reaction: dissociation of double molecules. ΔH = half heat of reaction	$C_i=\boldsymbol{R}\left(\frac{\Delta H}{\boldsymbol{R}T}\right)^2\frac{\exp(-\Delta H/\boldsymbol{R}T)}{\{1+[\frac{1}{2}\exp(-\Delta H/\boldsymbol{R}T)]^2\}^{3/2}}$	(15)
(3)	Two-state reaction: excitation of degenerate vibrational level. $\Delta H=\Delta E=\boldsymbol{h}\nu$ n = degeneracy; ν = frequency	$C_i=n\boldsymbol{R}\left(\frac{\Delta E}{\boldsymbol{R}T}\right)^2\frac{\exp(-\Delta E/\boldsymbol{R}T)}{\{1+n\exp(-\Delta E/\boldsymbol{R}T)\}^2}$	(14)
(4)	Excitation of vibration: all energy levels contributing to C_i.	$C_i=n\boldsymbol{R}\left(\frac{\Delta E}{\boldsymbol{R}T}\right)^2\frac{\exp(-\Delta E/\boldsymbol{R}T)}{\{1-\exp(-\Delta E/\boldsymbol{R}T)\}^2}$	(13)

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* See Appendix, however, for alternative conditions leading to a similar result.

§ 3. INTERPRETATION OF EXPERIMENTAL RESULTS

3.1. *Curve Fitting*

The theoretical formulae (16), (17) and (18) which have been derived from the general equation (11) under different assumptions are all of the same form

$$\beta_i/\beta_0 = QC_i \qquad \ldots\ldots(19)$$

where Q is a factor depending on temperature, and has the values $\gamma/(\gamma-1)C_p$, $(\gamma-1)/C_p$ and $1/C_p$ for the three cases, respectively. The left-hand side of equation (19) is obtained from the experimental measurements by means of equation (7) and it will also be dependent upon temperature in a known way. The temperature dependence of C_i, however, is controlled by the value of ΔH (equation (12)), which is unknown. The following graphical procedure has been developed for the purpose of testing the validity of the theoretical relations (16), (17) and (18) against experimental results.

The problem to be solved is essentially this: the value of $\beta_i/\beta_0 Q$ is known from experimental data for a range of temperatures, and it is suspected that C_i must be of the form

$$C_i(m, n, \Delta H) = m\boldsymbol{R}\left(\frac{\Delta H}{\boldsymbol{R}T}\right)^2 \frac{\exp(-\Delta H/\boldsymbol{R}T)}{\{1+n\exp(-\Delta H/\boldsymbol{R}T)\}^2} \qquad \ldots\ldots(20)$$

where n can only have the values -1, $+1$, $+2$, $+3$, ... but ΔH is unknown. It can be seen that $m=n=1$ gives equation (12), $m=1$, $n=-1$ gives equation (13) and $m=n$ gives equation (14). (It is found that the dissociation equation (15) is practically identical with equation (12) in the range of chief interest.) Now, if $p=\log_{10} C_i$ is plotted as a function of $q=\log_{10} T$ for an artibrary value ΔH_0 of ΔH (Figure 1), then moving the graph in the p-direction by an amount $\log_{10} a$ is equivalent to increasing m to am; similarly shifting the graph in the q-direction by an amount $\log_{10} b$ is equivalent to multiplying ΔH_0 by b. Hence, if the value of $\log_{10} \beta_i/\beta_0 Q$ is plotted on graph paper as a function of $\log_{10} T$, and curves of $\log_{10} C_i(m, n, \Delta H_0)$ are plotted, to the same scale, on a transparent celluloid sheet for the values $\Delta H_0 = 4$ kcal/mole; $m=n=1$; $m=-n=1$;

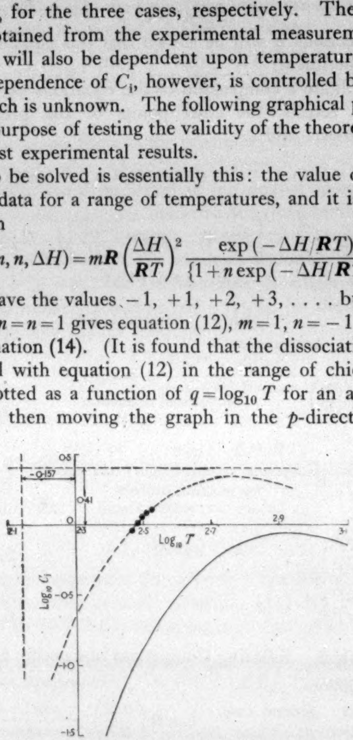


Figure 1. Example of curve fitting for acetic acid.

● Experimental values.

Full curve represents $C_1 \doteq R\left[\frac{\Delta H}{RT}\right]^2 \frac{\exp(\Delta H/\mathbf{R}T)}{\{1+\exp(-\Delta H/\mathbf{R}T)\}^2}$ for $\Delta H = 4$ kcal. per mole.

Values derived from fitted curve (— — —) are $m=2{\cdot}6$ (=antilog$_{10}$ 0·41), $\Delta H = 2{\cdot}8$ kcal/mole (=4/antilog$_{10}$ 0·157).

$m=1$, $n=2$; $m=1$, $n=3$; etc., then, by sliding the transparent curves over the graph paper, positions for coincidence of the experimental ($\log_{10}\beta_i/\beta_0 Q$) and theoretical ($\log_{10} C_i$) curves can be found. These positions will differ from the reference locations, where the axes coincide, by amounts $\log_{10} a$ and $\log_{10} b$, which will give the required numerical quantities: $m=a$, $\Delta H=b\Delta H_0=4b$ kcal/mole. The value of m, however, must be unity, except when the curves for $n=2, 3, 4, \ldots$ are used, and then m and n must be equal. *The value of m is, therefore, a criterion for deciding whether the experimental results do, in fact, confirm the theory.* Figure 1 indicates the principle of the method outlined here and shows the experimental values of $C_p(\beta_i/\beta_0)$ for acetic acid coinciding with a portion of the curve

$$C_i=\boldsymbol{R}\left(\frac{\Delta H}{\boldsymbol{R}T}\right)^2 \frac{\exp(-\Delta H/\boldsymbol{R}T)}{\{1+\exp(-\Delta H/\boldsymbol{R}T)\}^2}$$

plotted as described above. It can be seen that $\log_{10} a=0{\cdot}41$ giving $a=m=2{\cdot}6$; $\log_{10} b=-0{\cdot}16$ giving $\Delta H=b\Delta H_0=4/1{\cdot}44=2{\cdot}8$ kcal/mole.

3.2. *Acetic and Propionic Acids*

In order to apply the method of curve fitting described in the previous section, it is necessary to calculate from the measurements the values of the relaxing specific heat over the available temperature range corresponding to the three special cases given in Table 1 and expressed by equations (16), (17) and (18) respectively. Equation (17) given by Herzfeld is, however, an approximation for the condition $C_i \ll C_p$, and in what follows the exact equation

$$\beta_i/\beta_0=(\gamma-1)C_i/(C_p-C_i) \text{ or } C_i=C_p/\{1+(\gamma-1)\beta_0/\beta_i\} \quad \ldots\ldots(17\text{ a})$$

will be used.

The values calculated from the experimental data for acetic and propionic acids are given in Table 3, those of $\gamma=C_p/C_v$, being obtained from the formula $\gamma-1=\epsilon^2c^2T/C_p$ where c is the measured velocity of sound and the values of C_p and the expansion coefficient ϵ are taken from the International Critical Tables (1928).

The results given in Table 3 are now plotted in the manner described in § 3.1 and the method of curve fitting is employed to determine consistent values of m, n and ΔH, appearing in equation (20). Taking, first, $n=1$, the values of m and ΔH derived for each special case (Table 1) are given in Table 4.

Table 3

T(°c.)	μ_m	C_p (cal/mole/°c.)	γ	$C_i=\dfrac{2\mu_m}{\pi}\dfrac{(\gamma-1)}{\gamma}C_p$ (eqn. (16))	$C_i=\dfrac{C_p}{1+\frac{1}{2}\pi(\gamma-1/\mu_m)}$ (eqn. (17 *a*))	$C_i=\dfrac{2\mu_m}{\pi}C_p$ (eqn. (18))
	Acetic acid					
20	0·0510	29·25	1·214	0·168	3·855	0·951
30	0·0545	29·80	1·208	0·178	4·255	1·034
40	0·0583	30·38	1·202	0·190	4·725	1·132
50	0·0610	30·92	1·196	0·197	5·113	1·202
60	0·0641	31·50	1·190	0·205	5·570	1·287
	Propionic acid					
8	0·0107	32·97	1·272	0·0481	0·806	0·225
21	0·0134	33·11	1·265	0·0593	1·033	0·283
31	0·0154	33·20	1·260	0·0673	1·207	0·326
41	0·0178	33·31	1·253	0·0764	1·428	0·378
51	0·0201	33·42	1·246	0·0846	1·652	0·428

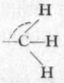
Table 4

$$C_{\mathrm{i}}=mR\left(\frac{\Delta H}{RT}\right)^2\frac{\exp(-\Delta H/RT)}{\{1+\exp(-\Delta H/RT)\}^2}\,;\qquad \Delta H=\Delta E+P^*\Delta V$$

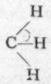
		Case (a) $\triangle E=0$ $C_{\mathrm{i}}=\frac{2\mu_{\mathrm{m}}}{\pi}\frac{\gamma-1}{\gamma}C_{\mathrm{p}}$	Case (b) $\Delta V=0$ $C_{\mathrm{i}}=\frac{C_{\mathrm{p}}}{1+\frac{1}{2}\pi(\gamma-1)/\mu_{\mathrm{m}}}$	Case (c) $\Delta V=0$ $C_{\mathrm{i}}=\frac{2\mu_{\mathrm{m}}}{\pi}C_{\mathrm{p}}$
Acetic acid	m	0·29	15	2·6
	ΔH	2·3	3·2	2·8
Propionic acid	m	0·33	13	2·3
	ΔH	3·5	4·2	3·8

According to the previous analysis the calculated value of *m* in Table 4 should be unity in case (*a*), and where it is other than unity in cases (*b*) and (*c*) agreement is only possible if a degeneracy be assumed. It has already been mentioned that the value of *m* in equation (20) is a criterion for deciding which theoretical approach is applicable. In the process of curve fitting the value of *m* which is obtained must equal unity or the assumed value of *n*, but in case (*b*) it is found that taking a larger value of *n* only increases *m*. However, the values of *m* for both liquids under case (*c*) suggest that the excitation of a vibrational energy level may account for the heat of reaction, $\Delta H=\Delta E$; in such a case a degeneracy of the order of three would be consistent with a vibration of carbon–hydrogen bonds. The closest agreement between the derived value of *m* and the assumed value of *n* is obtained by taking $n=3$ for both acetic and propionic acids: the values then obtained are given in Table 5. The correspondence between the frequencies seen in Table 5 and the two frequencies for the bending of carbon–hydrogen bonds given by Herzberg (1947), namely:

(1) $\nu_1\simeq 1{,}000\ \mathrm{cm}^{-1}$



(2) $\nu_2\simeq 1{,}450\ \mathrm{cm}^{-1}$



shows that these are probably connected with the process responsible for the observed behaviour, and further, the degeneracy of three is reasonable. It is concluded, therefore, that the measured dispersions in acetic and propionic acids can be interpreted on the basis of a relaxation of the transfer of energy between the ground state and a vibrational energy level corresponding to bending of the C–H bonds. Different bending modes are responsible for the behaviour of the two liquids, that for acetic acid being given in diagram (1) above ($\nu_1\simeq 1{,}000\ \mathrm{cm}^{-1}$), and that for propionic acid by diagram (2) ($\nu_2\simeq 1{,}450\ \mathrm{cm}^{-1}$). Moreover, the analysis given by the authors with assumption (*c*) in Table 1 leads to self-consistent results, which is not so for assumptions (*a*) and (*b*).

Table 5

$$C_{\mathrm{i}}=mR\left(\frac{\Delta E}{RT}\right)^2\frac{\exp(-\Delta E/RT)}{\{1+3\exp(-\Delta E/RT)\}^2}\,;\qquad \Delta E=h\nu$$

	n	m	ΔE(kcal/mole)	Frequency of vibration (ν cm^{-1})
Acetic acid	3	3·0	2·9	1020
Propionic acid	3	2·8	4·0	1400

It has been assumed that the value of C_p to be substituted in equation (18) is the static specific heat, which may not be strictly correct if other relaxations occur at lower frequencies. However, as can be seen by reference to columns 3 and 7 of Table 3, it is unlikely that these would contribute a sufficient amount to the total specific heat to cause much difference in the values derived above.

3.3. *Carbon Disulphide*

Measurements of the absorption in carbon disulphide are shown in Figure 2, these being a combination of results obtained in this laboratory (Lamb and Andreae 1951) at frequencies up to 70 Mc/s. with some by Rapuano (1950) at frequencies greater than 75 Mc/s. The curve drawn is based on the authors' measurements and is of the form

$$\alpha/f^2 = \frac{A}{1+(f/f_c)^2} + B \qquad \ldots\ldots(21)$$

with $A = 5{,}385 \times 10^{-17}$ sec^2 cm^{-1}, $B = 428 \times 10^{-17}$ sec^2 cm^{-1}, $f_c = 72{\cdot}1$ Mc/s. The first term of (21) is characteristic of a single relaxation process and the corresponding expression for μ the absorption per wavelength, due to the process is $\mu = (Ac/2\pi\bar{\tau})\omega\bar{\tau}/(1+\omega^2\bar{\tau}^2)$ where $2\pi\bar{\tau} = 1/f_c$, $\bar{\tau}$ being the relaxation time and f_c the 'characteristic frequency'. Hence $\mu_m = \frac{1}{2}Acf_c = \frac{1}{2}\pi\beta_i/\beta_0$ from (7). Allowing for a velocity dispersion of 7%, since $\mu_m/\pi \simeq 0{\cdot}07$, and taking Huddart's (1950) value for the velocity at low frequencies ($c_0 = 1{\cdot}14 \times 10^5$ cm/sec. at 25° c.) gives the experimental value for μ_m as 0·23. In a non-polar liquid such as carbon disulphide it is unlikely that there will be any appreciable order among the molecules at temperatures well above the melting point (—108·6° c.), so that the existence of an equilibrium between, say, an ordered and a random arrangement of the molecules is discounted; further, the molecules are stable so that an equilibrium of dissociation is improbable. The most likely cause of the observed behaviour, therefore, is to be found in an equilibrium between molecules which have been excited to a higher energy level and those which are in the ground state. The relaxation times to be expected from the excitation of rotational energy levels are too small to cause ultrasonic dispersion in the neighbourhood of 70 Mc/s. (Figure 2). It remains to examine equilibria between molecules excited to higher vibrational levels and those in the ground state.

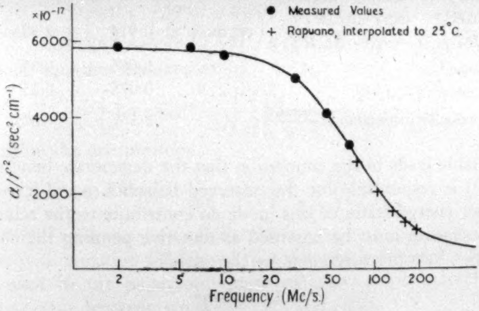


Figure 2. Absorption in carbon disulphide.

There are four normal vibrations of the carbon disulphide molecule, two being degenerate: these and the corresponding fundamental frequencies given by Herzberg (1947) are shown in Table 6. Assuming a two-state reaction it is possible from equation (14) to calculate the minimum relaxing component $C_{\mathrm{i(min)}}$ which each mode can contribute to the specific heat ($n=2$ for the degenerate vibration and $n=1$ for the other two modes). Likewise the maximum possible contribution $C_{\mathrm{i(max)}}$ is given by equation (13), where it is assumed that all the higher energy states of each vibration relax at the same rate. Substitution of these values in equation (18) gives the minimum and maximum values of μ_{m} associated with each mode, assuming that the authors' equation (18) represents the condition $\Delta V=0$ (Table 1, case (*c*)). The results obtained in this manner are collected in Table 6. Comparison of the experimental value of μ_{m} ($=0{\cdot}23$) with the results given in the table leads to the conclusion that the degenerate bending vibration ($\nu_2=397\ \mathrm{cm}^{-1}$) is responsible for the observed behaviour, and it would appear that the higher energy states of this mode do contribute to the relaxing specific heat; this conclusion must be regarded as tentative pending the completion of variable temperature measurements by the authors.

Table 6. Carbon Disulphide

$T=25^\circ$ c.; $C_{\mathrm{p}}=18{\cdot}37$ cal/mole/°c. (*International Critical Tables*)

Mode of vibration	1	2	3
Degeneracy, n	1	2	1
Frequency, $\nu\,(\mathrm{cm}^{-1})$	657	397	1523
$h\nu/RT=\Delta E/RT$	3·171	1·918	7·361
$K=\exp(-\Delta E/RT)$	0·042	0·147	0·00064
$C_{\mathrm{i(min)}}=nR\left(\dfrac{\Delta E}{RT}\right)^2\dfrac{\exp(-\Delta E/RT)}{\{1+n\exp(-\Delta E/RT)\}^2}$ cal/mole/°c.	0·772	1·282	0·0689
$C_{\mathrm{i(max)}}=nR\left(\dfrac{\Delta E}{RT}\right)^2\dfrac{\exp(-\Delta E/RT)}{\{1-\exp(-\Delta E/RT)\}^2}$ cal/mole/°c.	0·914	2·951	0·069
$\mu_{\mathrm{m(min)}}\simeq\frac{1}{2}\pi C_{\mathrm{i(min)}}/C_{\mathrm{p}}$	0·066	0·11	0·0059
$\mu_{\mathrm{m(max)}}\simeq\frac{1}{2}\pi C_{\mathrm{i(max)}}/C_{\mathrm{p}}$	0·078	0·25	0·0059
Max. possible velocity dispersion $\simeq\dfrac{\mu_{\mathrm{m(max)}}}{\pi}$	2·5%	8%	$<\frac{1}{4}$%

3.4. *Toluene*

Recent measurements by Moen (1951) have shown that the absorption in toluene is characteristic of a relaxation process. Moen's results cover the frequency region 150 kc/s. to 1·2 Mc/s. and, when taken together with measurements at higher frequencies obtained in this laboratory by Huddart (1950), make it possible to extrapolate the experimental value of μ_{m} corresponding to 27° c.: this is found to be $\mu_{\mathrm{m}}=3{\cdot}8\times10^{-5}$. By assuming, on the basis of previous success, the participation of a C–H bond *stretching* vibration (for which Herzberg (1947) gives $\nu\simeq2{,}960\ \mathrm{cm}^{-1}$) and calculating the expected value of μ_{m}, for a degeneracy of three, from equations (14) and (18) there is obtained the value $\mu_{\mathrm{m}}\simeq3{\cdot}2\times10^{-5}$. Within the limits set by the accuracy of Moen's measurements and the process of extrapolating the value of μ_{m}, the two values for μ_{m} show sufficient agreement to strengthen the hypothesis that the experimental behaviour is due to the relaxation of the transfer of energy to the stretching vibration of the C–H bonds in the methyl group of toluene.

It is hoped to extend these measurements in this laboratory and to study the behaviour at different temperatures. As in the case of carbon disulphide, it is essential to know the temperature dependence of μ_m for toluene before any confidence can be placed in the suggestions put forward for this liquid.

ACKNOWLEDGMENTS

The authors are grateful to Professor Willis Jackson for the support which he has given to this work. One of them (J. H. A.) wishes to acknowledge a Maintenance Grant from the Department of Scientific and Industrial Research, and the award of a Ferranti Scholarship by the Institution of Electrical Engineers.

APPENDIX

It can be shown that the assumptions underlying Hall's isothermal expression (equation (16)) may be expressed in the form

$$\Delta T=0;\quad \frac{\Delta V}{\Delta P}=\left(\frac{\partial V}{\partial P}\right)_T=\left(\frac{\partial V}{\partial P}\right)_x \qquad \ldots\ldots(\mathrm{A\,1})$$

while the authors' assumptions $\Delta V=0;\quad \frac{\Delta P}{\Delta T}=\left(\frac{\partial P}{\partial T}\right)_S \neq \left(\frac{\partial P}{\partial T}\right)_x$ ......(A 2)

do not exhibit a corresponding symmetry. However, if the compressibility is expressed in an alternative form

where $$\beta=-\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_S\left[\left(\frac{\partial T}{\partial n_2}\right)_y\left(\frac{\partial n_2}{\partial P}\right)_S+\left(\frac{\partial T}{\partial y}\right)_{n_2}\left(\frac{\partial y}{\partial P}\right)_S\right], \qquad \ldots\ldots(\mathrm{A\,3})$$

$$\beta_\infty=-\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_S\left(\frac{\partial T}{\partial y}\right)_{n_2}\left(\frac{\partial y}{\partial P}\right)_S, \qquad \ldots\ldots(\mathrm{A\,4})$$

then the final equation reduces to

$$\frac{\beta_i}{\beta_0}=\left[1+\gamma\beta_0 P^*\frac{V\Delta P}{\Delta H}\right]\left[1-\frac{\gamma}{\gamma-1}\frac{P^*\Delta V}{\Delta H}\right]\frac{C_i}{C_p}. \qquad \ldots\ldots(\mathrm{A\,5})$$

In this equation the assumptions

$$\Delta V=0;\quad \frac{\Delta P}{\Delta T}=\left(\frac{\partial P}{\partial T}\right)_V=\left(\frac{\partial P}{\partial T}\right)_y \qquad \ldots\ldots(\mathrm{A\,6})$$

lead to the result $\beta_i/\beta_0=\mathrm{C}_i/C_v$. ......(A 7)

Comparison with equation (18) shows that this result does not differ much from the result used in the paper and more extensive experimental data will be necessary to decide between equations (18) and (A 7).

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Cavitation Produced by Ultrasonics: Theoretical Conditions for the Onset of Cavitation

By E. A. NEPPIRAS and B. E. NOLTINGK
Mullard Research Laboratory, Redhill, Surrey

MS. received 23rd July 1951

ABSTRACT. An attempt is made to define theoretically the conditions for the appearance of cavitation in liquids subjected to alternating pressure changes. It is found that cavitation is restricted to a definite range of variations of the following parameters: (i) alternating pressure amplitude, (ii) frequency of the pressure wave, (iii) radius of the bubble nucleus, (iv) hydrostatic pressure. Under certain conditions, the change from non-cavitating to cavitating conditions is found to be exceedingly sharp, and in these cases it is shown that the threshold for cavitation can be accurately expressed in very simple terms.

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§ 1. INTRODUCTION

In a previous paper by the authors (1950), a theory is presented which describes the motion of a gas-filled cavitation bubble in a liquid subjected to alternating pressure changes. In this earlier work, the differential equation giving the motion of the bubble surface was solved for various arbitrary values of the parameters representing the size of the cavitation nucleus and the frequency and amplitude of the alternating pressure changes in the medium. The data obtained from these solutions were used to form an estimate of the intensity of cavitation action and its dependence on these parameters. It was found that this cavitation intensity, however it is measured, is very dependent on the nuclear size and that for any assumed nuclear radius an upper threshold in frequency must exist above which the conditions for true cavitation can never be realized. An increase in frequency for any given bubble size will result in the changes in bubble radius becoming less violent and finally degenerating into comparatively gentle oscillations as the frequency is increased above the resonant value of that particular size.

A further series of deductions has now been made from the original theory, and we find that cavitation is restricted, not only to a finite range of frequencies ω and nuclear sizes, radius R_0, but also to a fixed range of variations of hydrostatic pressure P_A and alternating pressure amplitude P_0. In the present theoretical work, an attempt is made to define the conditions (in terms of the four parameters ω, R_0, P_A, P_0) under which true cavitation can be said to occur. Under any other conditions it is found that the bubbles undergo forced oscillations, of comparatively small amplitude, at the frequency of the impressed pressure field. The change from non-cavitating to cavitating conditions may occur abruptly or quite slowly.

Some of the theoretical results obtained are of special importance. For example, in the previous work we were forced to accept the possibility that indefinitely small bubbles might serve as cavitation nuclei, but we now find a definite lower threshold in R_0 below which cavitation does not occur, and this threshold is, in general, extremely sharply defined and easily calculated in terms of P_0. This limitation in R_0 sets, in turn, an upper limit to all theoretical estimates of the intensity of cavitation, however this is expressed. It is also found possible to deduce an absolute threshold in P_0 for any assumed ω and P_A below which cavitation can never occur whatever the distribution of nuclear sizes; the importance of this result lies in the fact that R_0 is the only one of the four parameters mentioned above which is, in most practical cases, not fixed by the experimental conditions. The theoretical result therefore applies to any ordinary 'gassy' liquid such as would normally be used in practice as a medium for transmitting the pressure wave. A further, rather unexpected, result is the appearance of a theoretical upper threshold in P_0, independent of frequency, above which cavitation is suppressed.

§ 2. CAVITATION THRESHOLDS

(i) *Thresholds for R_0 and ω*

The excess pressure due to surface tension demands that the pressure of gas in very small bubble nuclei is extremely high, and it becomes difficult to see how such bubbles can exist for long even in a liquid which is saturated with gas under the existing conditions. For this reason, in the theoretical work already quoted, a somewhat arbitrary restriction was placed on the smallness of the cavitation nuclei considered. However, recent experimental work (for example, Blake 1949) suggests that extremely small bubbles with radii of approximately $0{\cdot}1\,\mu$ are in fact stabilized in ordinary liquids, although the method of stabilization is not known. We have therefore extended our theoretical survey down to much smaller nuclear radii, and we find that for R_0 sufficiently small, cavitation does not occur because surface tension forces prevent the nucleus from ever growing large enough.

For example, radius–time curves have been obtained similar to that shown in Figure 1 of the previous work, corresponding to $\omega = 9 \times 10^4$, $P_0 = 4 \times 10^6$; $P_A = 10^6$ C.G.S. units and a wide range of nuclear sizes extending from $16\,\mu$ to $0{\cdot}15\,\mu$. We find that the curve for $R_0 = 0{\cdot}15\,\mu$ hardly rises above the time axis, while for $R_0 = 0{\cdot}4\,\mu$ the maximum radius R_m is only a few per cent less than that for R_0 very large. The relation between R_m and R_0 is given in Figure 1;

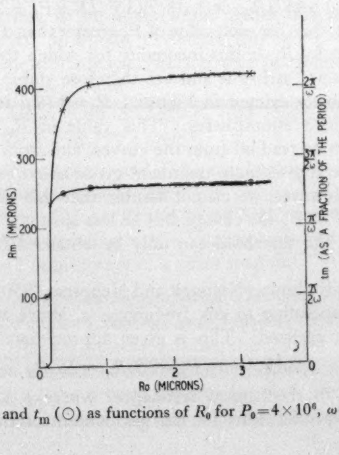

Figure 1. R_m (×) and t_m (○) as functions of R_0 for $P_0 = 4 \times 10^6$, $\omega = 9 \times 10^4$, $P_A = 10^6$.

R_m occurs at time t_m which is also shown plotted as a function of R_0. The curve for R_m shows a well-defined threshold in R_0 below which the bubbles do not expand significantly and therefore cavitation will not occur. In this region the motion is controlled by surface tension, and the changes in radius and radial velocity over the cycle are quite small; we can infer from this that the spatial variations in fluid pressure round the bubble during its motion will also be insignificant, so that the liquid tension at a great distance from the bubble at any instant, $P_A + P_0 \sin \omega t$, is almost the same as that in the liquid just outside the bubble surface. Under these simplified conditions we can readily deduce an approximate expression for the threshold R_0 as a function of the pressure amplitude. To this end we examine the conditions for the stable equilibrium of the bubble surface.

The pressure at any time just outside the bubble is given by

$$p = (P_A + 2S/R_0)(R_0/R)^3 - 2S/R \qquad \ldots\ldots(1)$$

where S is the surface tension of the liquid, assuming the gas changes to be isothermal. From this equation, it follows that p is a minimum with respect to R when $dp/dR = 0$, namely when $R = R_T = [3R_0^3(P_A + 2S/R_0)/2S]^{1/2}$. That this value of R makes p, defined by (1), a minimum may be easily verified. The alternative expression for p, obtained by neglecting inertia forces, is that for the pressure in the liquid well away from the bubble, namely $P_A + P_0 \sin \omega t$, and unless the minimum value of p, determined by (1), is greater numerically than the minimum of $P_A + P_0 \sin \omega t$, or $P_0 - P_A$, we have an unstable situation where the gas pressure and surface tension forces cannot be balanced by the pressure in the liquid just outside the bubble. Inertia terms must now be included and a rapid expansion of the bubble occurs. On the other hand, if the minimum p allowed by (1) is greater than $P_0 - P_A$ the bubble radius does not vary much from its initial value, and throughout its oscillations the gas pressure and surface tension forces are substantially in continuous equilibrium with the applied pressure in the liquid. The bubble radius is not stable in the sense that it fluctuates wildly if $(P_0 - P_A) > (P_A + 2S/R_0)(R_0/R_T)^3 - 2S/R_T > 0$ or, substituting for R_T, if $(P_0 - P_A) > 4S/3R_T$ or $> (8/9)[3S^3/2R_0^3(P_A + 2S/R_0)]^{1/2}$. If P_A is regarded as constant, then for each value of P_0 greater than P_A there is a minimum size of bubble given by R_0 in this inequality for which the bubble is unstable. Smaller bubbles do not satisfy it and are therefore stable. It may be verified that for the conditions depicted in Figure 1 $R_0 = 0{\cdot}19\,\mu$ is the threshold value for $P_0 - P_A$ equal to 3 atmospheres. This value of R_0 is very close to the critical R_0 which can be read off from the curves, and from this we infer that, in this case, the simple approximate treatment given above is sufficiently accurate for the purpose. However, we cannot assume that this will be so in all cases: for lower values of $P_0 - P_A$ the change will be less abrupt. In the general case, a reliable estimate of the threshold can only be obtained from the relevant set of (R, t) curves.

As we have already shown (Noltingk and Neppiras 1950), an upper threshold for R_0 exists, corresponding to any frequency ω, above which true cavitating conditions are never realized. This is given approximately by the equation

$$\rho\omega^2 R_0^2 = 3\gamma(P_A + 2S/R_0) - 2S/R_0 \qquad \ldots\ldots(2)$$

which corresponds to mechanical resonance, where ρ is the liquid density and γ the ratio of specific heats for the gas content of the bubble. As R_0 is increased above this value, the bubble executes a complex oscillation while the changes in R_0 remain comparable with R_0; but as R_0 is still further increased, the oscillations become more nearly sinusoidal.

We therefore see that both for bubbles such that R_0 is less than the lower threshold and for those for which R_0 is much greater than that given by equation (2) the changes in bubble radius are relatively small. Under these simplified conditions, the equation for the bubble motion reduces to a particularly simple form. This can be found by making the substitution $R=R_0+\Delta R$ in (1) and solving for ΔR on the assumption that $\Delta R/R_0$ is so small that powers higher than the first can be ignored. We then obtain

$$R=R_0-\frac{R_0P_0\sin\omega t}{3(P_A+4S/3R_0)}. \qquad \ldots\ldots(3)$$

R therefore varies sinusoidally with t with the same period as the impressed pressure but 180° out of phase with it. If ΔR is less than R_0 but higher order terms than the first power of $\Delta R/R_0$ cannot be ignored, we may readily obtain an expression for R of the form $R=R_0-a_1\sin\omega t+a_2(\sin\omega t)^2\ldots$, where $a_1, a_2, \ldots$ are functions of R_0, P_0 and P_A. The (R, t) curve is no longer sinusoidal but is still periodic with period $2\pi/\omega$.

Summarizing, therefore, we see that, as R_0 is increased from indefinitely small values, the changes in radius are at first small and the bubble motion is truly sinusoidal but 180° out of phase with the impressed pressure. The (R, t) curve becomes distorted as the lower threshold for R_0 is approached, and at the critical R_0 (corresponding to the particular pressure amplitude considered), a sudden expansion occurs, representing the start of true cavitation. Cavitation conditions continue until R_0 reaches and passes the resonant value (corresponding to the particular frequency considered) given by (2), after which the bubble motion again becomes sinusoidal and is again given by the simple equation (3).

(ii) *Absolute Thresholds for P_0 and ω*

We see, then, that there must exist a finite range of nuclear radii suitable for cavitation. As the frequency increases at any pressure amplitude, the range is reduced by a lowering of the upper limit. After this has been reduced to coincide with the lower threshold, cavitation can never occur, theoretically, whatever the distribution of nuclei. This is true for frequencies greater than that given by the equations (2) and

$$4S/3R_T=(8/9)[3S^3/2R_0^3(P_A+2S/R_0)]^{1/2}=K$$

or

$$2SR_T^2=3R_0^3P_A+6SR_0^2. \qquad \ldots\ldots(4)$$

We obtain the relation between ω and K by eliminating R_0 between (2) and (4) and making the substitution $R_T=4S/3K$.

The threshold frequencies are found to be high even for moderate values of K. For example, corresponding to $K=2$ atmospheres, we obtain $\omega/2\pi=$ about 10 Mc/s., taking $P_A=1$ atmosphere in a water medium.

(iii) *Limiting Cavitation Pressures*

We have previously shown (Noltingk and Neppiras 1950) that the maximum liquid pressures and velocities generated during the collapse of cavitation bubbles are very dependent on the size of the bubble nucleus, increasing rapidly as R_0 decreases. According to the present theory, however, there is a limit to the smallness of possible cavitation nuclei and this, in turn, sets a theoretical upper limit to the magnitude of the collapse pressures, etc. For example, we have previously shown that the maximum liquid pressure during collapse is given by $\{P^4/81(P_A+2S/R_0)\}(R_m/R_0)^9$ in terms of R_m and R_0, where P is the effective liquid pressure at $r=\infty$ during the collapse. Since R_m is a function of R_0, for a given P_0 and ω there will be an optimum R_0 at which the cavitation pressure is a maximum. Obviously, this R_0 is almost exactly that for which R_m/R_0 is a maximum, the variation of P with R_0 being quite slow. If therefore in Figure 1 we draw a tangent from the origin to the curve, the R_0 coordinate of the point of contact is the required optimum value. It is very slightly greater than the threshold R_0', as we would expect.

(iv) *Thresholds for P_A and P_0*

We turn now to the effect on the bubble motion of changes in the parameters P_A and P_0. It is not immediately obvious how the rapidity of the collapse of a cavitation bubble will be affected by changes in the standing pressure P_A. The bubbles will certainly grow larger if P_A is decreased, but the collapse will occur at a lower effective hydrostatic pressure, and these changes affect the cavitation intensity in opposite directions. But if the bubbles become very large the time τ required for their collapse may be so great that the pressure begins to go negative before the completion of the collapse, which will then become less violent. This will be true for $t_m+\tau>\omega^{-1}(2\pi+\sin^{-1}P_A/P_0)$. A lower threshold must therefore exist for P_A below which true cavitating conditions do not obtain owing to the bubble not having time to collapse completely. Reduction of P_A below this limit will result in slower changes in the bubble radius. This prediction has been verified by obtaining solutions of the equation of motion corresponding to P_A small compared with P_0. Strictly speaking, for bubbles which grow so large, it will not be permissible to ignore the effects of liquid vapour evaporated into the bubble during the motion as was done in the derivation of the equation of motion. The graphs of Figure 2 show the variation of R_m and p_m (the maximum collapse pressure) with P_A for the case $\omega=9\times10^4$, $P_0=4\times10^6$, $R_0=1{\cdot}6\times10^{-4}$ C.G.S. units. p_m was calculated on the assumption that the bubbles at maximum radius contain vapour at pressure always taken to be 10^4 dyne/cm^2 (about 0·75 cm. Hg), the vapour content

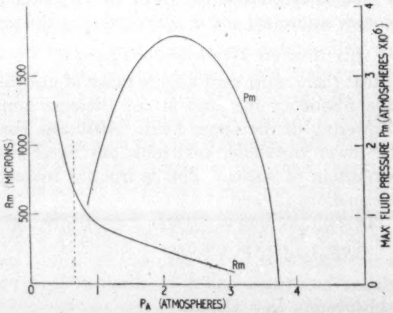

Figure 2. R_m and maximum fluid pressure as functions of P_A for $P_0=4\times10^6$, $\omega=9\times10^4$, $R_0=1{\cdot}6\times10^{-4}$.

remaining constant during the collapse. The effective liquid pressure P at which the collapse occurs has been estimated by a process of successive approximation, P finally being the mean liquid pressure at $r=\infty$ over the period t_m to $t_m+\tau$. The broken line indicates the limiting value of P_A below which the collapse will not be completed during the time that the external liquid pressure remains positive. We see that the peak of p_m occurs at a value of P_A greater than one atmosphere. The curve shows both upper and lower thresholds in P_A outside which cavitation pressures will not be developed during the motion. It must be remembered that these curves refer to particular values of the parameters ω, P_0 and R_0, so that only very limited conclusions can be drawn from them.

Solutions of the equation of motion have also been obtained corresponding to $P_A \geqslant P_0$. They resemble, in all respects, the (R, t) curves for P_0 less than the threshold pressure, as we would expect, for, unless P_0 is greater than P_A, P_0 is always less than the threshold whatever R_0 may be. When P_0/P_A is small, the equation of motion is, of course, given by (3). As an example, the curves of Figure 3 show the variation of R_m and t_m with P_0 for $P_A=10^6$, $\omega=9\times10^4$, $R_0=3\cdot2\times10^{-4}$ C.G.S. units. The sudden expansion of the bubble occurs at $P_0 \simeq P_A$. We see, then, that cavitation can be suppressed either by reducing P_0 to be less than P_A, approximately, or by increasing P_A to be greater than P_0, even in ordinary 'gassy' liquids containing comparatively large bubbles. These theoretical conditions correspond to well-known experimental facts.

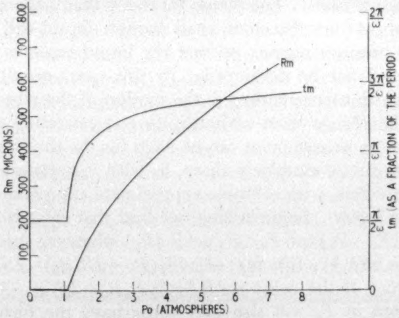

Figure 3. R_m and t_m as functions of P_0 for $R_0=3\cdot2\times10^{-4}$, $\omega=9\times10^4$, $P_A=10^6$.

It was noted above that decreasing P_A relative to P_0 may result in the collapse time τ becoming so large that the external pressure has become negative at $t=t_m+\tau$, and this implies a lower threshold in P_A. It also implies an upper threshold in P_0 if P_A is thought of as constant. For example, the curve of Figure 4 gives $t_m+\tau$ as a function of P_0 for the case $\omega=9\times10^4$, $P_A=10^6$, $R_0=3\cdot2\times10^{-4}$ C.G.S. units, the dotted line showing the points where $t_m+\tau$ becomes approximately equal to $\omega^{-1}(2\pi+\sin^{-1}P_A/P_0)$. At this point, $P_0 \simeq 6\cdot5$ atmospheres. It can readily be seen from the data previously given (Noltingk and Neppiras 1950) that this critical value of P_0 should be independent of frequency if R_0 is well below the resonant value (making R_m very much greater than R_0); also, we may note that the critical P_0 will not be very dependent on R_0 if this is well inside the cavitation range of nuclear sizes, for R_m changes only slowly with R_0 in this range.

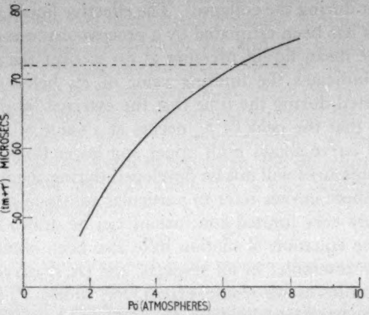

Figure 4. $t_{\mathrm{m}}+\tau$ as a function of P_0 for $R_0=3\cdot 2\times 10^{-4}$, $\omega=9\times 10^4$, $P_A=10^6$.

§ 3. CONCLUSION

We have shown that cavitation can be suppressed by sufficiently large alterations, in either direction, in any one of the three parameters P_A, P_0 and R_0 when the others remain fixed; ω also has an upper threshold, definable in terms of P_A and R_0. The theory indicates no lower threshold for ω and we should therefore expect that the cavitation intensity would continue to increase as ω was decreased. However, at sufficiently low frequencies the theory will cease to be applicable to practical cases. The reason for this is that if the radian frequency ω of the face of the generator becomes small enough, liquid will have time to flow towards the low pressure regions, so that the liquid tensions necessary for the bubble growth will not be developed. In this case, no alternating pressure field would be set up corresponding to the motion of the vibrator.

Sometimes the change from cavitating to non-cavitating conditions will be a gradual one and no precise point can be fixed for the threshold. On the other hand, the change can be extremely sharp, as with variations in P_0 for small R_0 (see Figure 1). In some cases, simple approximate expressions can be deduced for the cavitation limits. Summarizing, we find that cavitation will not occur: (i) for $R_T<4S/3(P_0-P_A)$ (or $P_0<P_A+4S/3R_T$) whatever the frequency, where $2SR_T^2=3P_AR_0^3+6SR_0^2$; (ii) for $\omega^2>[3\gamma(P_A+2S/R_0)-2S/R_0]/\rho R_0^2$ (or R_0 greater than a root of the cubic $\rho\omega^2R_0^3=3\gamma P_AR_0+2S(3\gamma-1)$) approximately, although variations in P_0 will also play some part, the formulae being more accurate the smaller P_0; (iii) for $P_A \geqslant P_0$ approximately, whatever ω and R_0 may be; (iv) for P_A so small (or P_0 so large) that the cavitation bubble has no time to collapse before the end of the pressure cycle.

Combining (i) and (ii) we obtain an absolute threshold for P_0, for any ω and P_A, below which cavitation can never occur whatever the distribution of nuclear sizes.

ACKNOWLEDGMENTS

This work was carried out under the auspices of the Mullard Radio Valve Company Ltd. The authors wish to thank the directors of the company for permission to publish, and Dr. C. F. Bareford, Manager of the Research Laboratory, for his encouragement in the project.

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Self-Magnetic Field in High Current Discharges

By M. BLACKMAN

Physics Department, Imperial College, London

MS. received 5th June 1951

ABSTRACT. The theory of the self-magnetic field in high current discharges, originally developed by Tonks, has been investigated omitting a special assumption used in the work of Tonks. The influence of the parameters describing the discharge conditions has been considered in some detail. A comparison is also made with a simple, though more approximate, version of the theory due to Thonemann and Cowhig. The two theories are shown to be in good agreement.

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§ 1. INTRODUCTION

The theory of the self-magnetic fields in high current discharges (the 'Pinch effect') has been given by Tonks (1939). This is based on the equations of Tonks and Allis (1937) governing the motion of electrons and positively charged ions under the action of electric and magnetic fields.

The equations were used in a cylindrical case in which the main current is in the x direction and a radial current flows to the walls. The theory is, however, limited by a special assumption: the walls are assumed to be conducting and so highly charged that there is a radial positive current but no electronic current.

We shall take, in this calculation, the more usual condition of a non-conducting wall. In this case, we should expect, in a stationary state, that the total current to the walls would be zero, i.e. the positive and negative currents cancel. We shall have to make some assumptions, in particular that the difference in density of the positive and negative charges is negligible compared with the magnitude of this density. Except in a certain limited region, this assumption will be shown to be consistent with the results of the calculations. Further, the charge density is taken to be small compared with that of the neutral molecules.

The basic theory is that due to Tonks (1939).

§ 2. BASIC EQUATIONS

As indicated above, we consider a cylindrical case, the main current being in the x direction, with a fixed electrostatic field $-\partial V/\partial x$ in this direction, i.e. $\partial V/\partial x$ is independent of r, where r is the radial cylindrical coordinate in the plane perpendicular to the x direction.

The equations for the components of the velocity of the electrons $\mathbf{u}_e$ and of the positives $\mathbf{u}_p$ (Tonks 1939) are

$$-\frac{(\mathbf{u}_r)_e}{D_e} = \alpha\left(\frac{\partial \eta}{\partial r} + \frac{1}{n_e}\frac{\partial n_e}{\partial r}\right) - \beta\xi \qquad \ldots\ldots(1)$$

$$-\frac{(\mathbf{u}_r)_p}{D_p} = \frac{1}{n_p}\frac{\partial n_p}{\partial r} - \frac{e}{kT_p}\frac{\partial V}{\partial r} \qquad \ldots\ldots(2)$$

$$-\frac{(\mathbf{u}_x)_e}{D_e} = \alpha\xi + \beta\left(\frac{\partial \eta}{\partial r} + \frac{1}{n_e}\frac{\partial n_e}{\partial r}\right) \qquad \ldots\ldots(3)$$

with
$$\left.\begin{aligned}\alpha=\frac{\gamma_a}{1+h^2}\,;\quad \beta=\frac{\gamma_b h}{1+h^2}\,;\quad h=\frac{3D_e}{(\overline{C}_e)^2}\,\frac{eH}{mc}\,;\\ \xi=\frac{e}{kT_e}\,\frac{\partial V}{\partial x}=\frac{\partial\eta}{\partial x}\quad\left(\eta=\frac{eV}{kT_e}\right);\\ D_e=\tfrac{1}{3}l_e\overline{C}_e\,;\quad D_p=\tfrac{1}{3}l_p\overline{C}_p\,;\end{aligned}\right\}\qquad\ldots\ldots(4)$$

where n_e = number of electrons per cm³, n_p = number of positives per cm³, e = the electronic charge, m = mass of the electron, c = velocity of light, $\overline{C}_e$ = mean velocity of the electrons, $\overline{C}_p$ = mean velocity of the positives, l_e = mean free path of the electrons, l_p = mean free path of the positives, k = Boltzmann's constant, T_e = absolute temperature of the electrons, T_p = absolute temperature of the positives, H = magnetic field.

The magnetic field $H=H_\varphi$ where ϕ is the cylindrical angle coordinate. From the symmetry of the problem, H_φ is the only component of the magnetic field different from zero; also the densities n_e and n_p and the potential V, as well as H, are functions of r only. The functions γ_a and γ_b are functions of h, given by Tonks (1939).

The magnetic field is related to $(u_x)_e$ by the expression

$$\frac{1}{r}\,\frac{d}{dr}(rH)=4\pi(u_x)_e\frac{n_e e}{c}=-\frac{4\pi n_e e}{c}D_e\left[\alpha\xi+\beta\left(\frac{d\eta}{dr}+\frac{1}{n_e}\,\frac{dn_e}{dr}\right)\right].\qquad\ldots\ldots(5)$$

A further relation which we shall need is the continuity equation

$$\frac{d}{dr}\{n_p(u_r)_p r\}=r\lambda n_e,\qquad\ldots\ldots(6)$$

where λ is a parameter which allows for the multiplication by collision.

In the solution of the equations we make the approximation $\gamma_a=1=\gamma_b$. This is a rough approximation, but it will be shown below that the results are not significantly different when γ_a and γ_b are introduced as functions of h. Further, we use the fact that $T_p\ll T_e$. It will be shown later that the results are largely independent of the ratio T_p/T_e.

Taking $n_e=n_p=n$ and $(u_r)_p=(u_r)_e=u_r$, we obtain u_r by eliminating $d\eta/dr$ from (1) and (2)

$$u_r=-\frac{D_eD_p}{\alpha T_pD_e+T_eD_p}\left[\alpha(T_e+T_p)\frac{1}{n}\,\frac{dn}{dr}-\beta\xi T_e\right].\qquad\ldots\ldots(7)$$

Using the continuity equation (6) to eliminate u_r we obtain with $T_p\ll T_e$

$$\begin{aligned}\frac{\lambda}{T_e}\left(\frac{T_p}{D_p}+\frac{T_e}{D_e}(1+h^2)\right)&=-\frac{1}{n}\,\frac{d^2n}{dr^2}+\frac{1}{n}\,\frac{dn}{dr}\left[h\xi-\frac{1}{r}+\frac{2hA}{1+A(1+h^2)}\,\frac{dh}{dr}\right]\\&\quad+\frac{\xi h}{r}+\frac{\xi}{1+h^2}\,\frac{dh}{dr}\left[1-h^2+\frac{2h^2}{1+A(1+h^2)}\right]\qquad\ldots\ldots(8)\end{aligned}$$

where
$$A=T_eD_p/T_pD_e.\qquad\ldots\ldots(9)$$

The equations for the magnetic field (5) in terms of h (see equation (4)) lead to the form

$$\frac{1}{r}\,\frac{d}{dr}(hr)=\gamma n\left[\alpha\xi+\beta\left(\frac{d\eta}{dr}+\frac{1}{n}\,\frac{dn}{dr}\right)\right]\qquad\ldots\ldots(10)$$

with
$$\gamma=-12\pi^2e^2D_e{}^2/mc^2(\overline{C}_e)^2.\qquad\ldots\ldots(11)$$

Eliminating η with the help of (7) and (10), we obtain

$$\frac{1}{\gamma}\frac{dh}{dr} = -\frac{1}{\gamma}\frac{h}{r} + \frac{\xi n}{1+h^2} + \frac{\xi n h^2}{(1+h^2)[1+A(1+h^2)]} + \frac{Ah}{1+A(1+h^2)}\frac{dn}{dr}. \quad \ldots\ldots(12)$$

Equations (8) and (12) are then the basic equations to be solved for n and h as a function of r.

These equations are solved by numerical methods. The boundary conditions at $r=0$ can be found from the limiting forms of the equations. These can be seen to be consistent (for small r) if

$$n = n_0(1-br^2), \qquad h = fr, \quad \ldots\ldots(13)$$

where

$$\left.\begin{aligned} f &= \tfrac{1}{2}\gamma n_0 \xi \\ 4b + 2f\xi &= \frac{\lambda}{T_e}\left(\frac{T_p}{D_p} + \frac{T_e}{D_e}(1+h^2)\right) = B \end{aligned}\right\} \quad \ldots\ldots(14)$$

and n_0 is the value of the charge density at $r=0$, of either the positive or the negative charge. The constant b is determined from the values of the parameters ξ, T_e, T_p, λ, γn_0.

§ 3. THE CASE OF $\lambda=0$

For the purposes of numerical calculation, the gas was taken to be hydrogen, and its density 10^{16} molecules per cm³. The electron temperature was taken to correspond to 20 ev. The value of γ is then $-3{\cdot}9\times 10^{-13}$ C.G.S. units. All other parameters were subject to some examination, the magnitudes of these parameters being given below (Table 1).

Table 1. Values of Parameters used

γn_0	$-3{\cdot}90$	$-39{\cdot}0$	-390
ξ	4·0; 8·0; 48·0	0·5; 1·125*; 2·0; 4·5	0·2; 0·5; 1·0
T_e/T_p	160	160, 12*	160

The values of γn_0 are obtained by choosing n_0 to have values of 10^{13}, 10^{14} and 10^{15} molecules per cm³. The set of values of γn_0 does, however, allow one to see the effect of various electron temperatures and of the density of the neutral gas (since γ depends on D_e^2). The numerical calculation depends only on n/n_0. The variation of n and of h with T_e/T_p was examined in one case $\xi=1{\cdot}125$, $T_e/T_p=12$, indicated in Table 1 by an asterisk. These calculations show a negligible change in $n(r)$ and $h(r)$; these are in fact too small to be shown in diagrams on the scale of Figures 1 and 2. It was, therefore, not thought necessary to extend the investigation of this parameter to other cases.

The $n(r)$ and $h(r)$ curves based on the parameters given in Table 1 are given in Figures 1 and 2. The $n(r)$ curves are expressed in the dimensionless form n/n_0. The $h(r)$ values are negative and can be converted to magnetic fields (in gauss) by multiplying $-h(r)$ by 26·3.

A study of these curves for different values of γn_0 shows further that both sets cannot simultaneously be reduced to one another by some scaling factor. The general features are as follows: (*a*) For a fixed value of γn_0 the maximum value of h is independent of the value of ξ. (*b*) An increase in the value of ξ (at fixed γn_0) makes the 'pinch' more marked, by which we mean the value of r at which $n(r)$ falls to its half value; this point is emphasized in Table 2. (*c*) A characteristic feature of all the $n(r)$ curves, which is apparent in the detailed calculations and which can also be seen in the curves, is the increase in the absolute value of $(1/n)dn/dr$ followed by a slow decrease. Since $(1/n)dn/dr$ is negative, the curves have the form of a decreasing exponential (as a function of r), after a certain value of r. (*d*) The $h(r)$ curves rise linearly at first, then more slowly to a maximum value, after which the decrease is gradual. This maximum value occurs at practically the same value of r as the maximum value of $|(1/n)dn/dr|$.

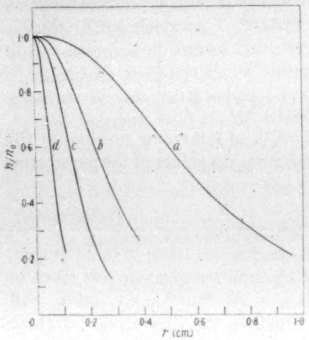

Figure 1. The radial variation of the electronic density.

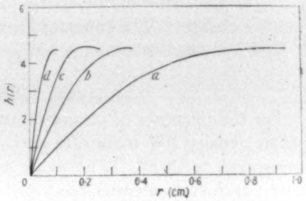

Figure 2. The variation of the magnetic field as a function of radius.

a: $\xi=0{\cdot}5$; b: $\xi=1{\cdot}125$; c: $\xi=2{\cdot}0$; d: $\xi=4{\cdot}5$. In all cases $\gamma n_0=-39{\cdot}0$ and $T_e/T_p=160$.

The values of $|h_{\max}|$ for $\gamma n_0=-390{\cdot}0$, $-39{\cdot}0$ and $-3{\cdot}9$ are 14·2, 4·5 and 1·4 respectively. The position of $|h_{\max}|$ as a function of r, for the various ξ values, can be obtained with fair accuracy by multiplying the r values in Table 2 by 1·60.

Table 2. 'Half value' Points in $n(r)$ Curves.

γn_0	$-390{\cdot}0$			$-39{\cdot}0$				$-3{\cdot}9$		
ξ	1·0	0·50	0·20	4·5	2·0	1·125	0·50	48	8	4
r (cm.)	0·090	0·175	0·455	0·065	0·145	0·330	0·560	0·019	0·116	0·222

We consider next the validity of two of the assumptions made in the calculation: (i) It is assumed that one can neglect the difference in density of the positively charged particles and of the electrons. This point can be checked by calculating $d\eta/dr$ (i.e. the electrostatic field) from (10), obtaining its derivative numerically and then inserting in Poisson's equation. In most cases, the value of $d\eta/dr$ was found to be too small to be calculated with any accuracy; but the upper limit of the difference in charge density is small enough to justify the assumption amply. (ii) To simplify the numerical work, it was assumed that γ_a and γ_b in (4) could be replaced by unity. This assumption has been examined in a typical case $\xi=2$, $T_e/T_p=160$. The calculations for $n(r)$ and $h(r)$ were carried through again, using the curves for $\gamma_a(h)$, $\gamma_b(h)$ given by Tonks (1939). Other curves for $d\gamma_a/dh$ and $d\gamma_b/dh$, which were needed, were constructed from these γ_a, γ_b functions. A comparison of these calculations, with the previous ones is made in Figure 3. It will be seen that the assumption made does not materially change the conclusions drawn from the results. The change in the curves is small, for example, compared with the effect of changing ξ from 2·0 to 4·5 or from 2·0 to 1·125.

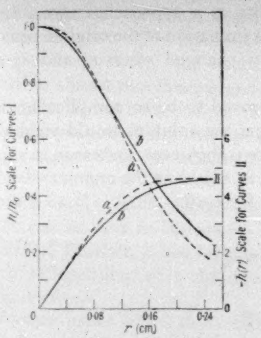

Figure 3. The radial variation of the electronic density (I) and magnetic field (II): comparison of the approximate with the more accurate calculation. In curve a, $\gamma_a = 1 = \gamma_b$, in curve b, γ_a and γ_b are treated as functions of h. The parameters are $\xi = 2{\cdot}0$, $\gamma n_0 = -39{\cdot}0$.

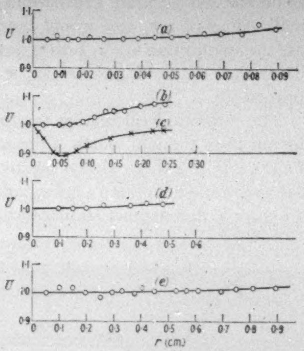

Figure 4. The radial variation of $U = (-u_x)/D_e\xi$. a: $\xi = 4{\cdot}5$; b: $\xi = 2{\cdot}0$; c: $\xi = 2{\cdot}0$; d: $\xi = 1{\cdot}125$; e: $\xi = 0{\cdot}5$. In c, γ_a and γ_b are treated as functions of h, in all other cases $\gamma_a = 1 = \gamma_b$. $\gamma n_0 = -39{\cdot}0$ in all cases.

We can further obtain from the $n(r)$, $h(r)$ curves the value of the electron velocity $(u_x)_e$ in the longitudinal direction. This is best done in the form $-u_x/D_e\xi$ which has the value unity at $r = 0$. This quantity was calculated for all the cases quoted in Table 1. It was found to be nearly independent of r, the maximum variation being of the order of 10%. A typical set of curves is shown in Figure 4. That the conclusion arrived at is independent of the assumption $\gamma_a = 1 = \gamma_b$ is shown in Figure 4 (c).

§ 4. THE CASE OF $\lambda \neq 0$

In the case $\lambda \neq 0$ the solutions of equations (8) and (12) do not show the same character as for $\lambda = 0$. This is illustrated in Figure 5 where the $n(r)$ curve is plotted (curve I). The parameters here are $\xi = 1{\cdot}125$, $B = 43{\cdot}2$ (cf. equation (14)), $T_e/T_p = 160$. The value of B is deduced from an ionization cross section of $0{\cdot}58 \times 10^{-16}$ cm^2 (which is about one-eighth of the collision cross section).

It will be seen that the $n(r)$ curve will cross the r axis (i.e. produce negative values of the density). This point has been checked in two independent calculations (for one of which we are indebted to Mr. K. Skwirzinski). The calculation has also been repeated with the more exact form of the differential equations obtained by dropping the approximation $\gamma_a = 1 = \gamma_b$. This alters the result only to a small extent; the curve shown is that obtained in this last calculation. That the type of curve obtained is independent of the choice of γn_0 and of ξ has also been checked by investigating two special cases.

It is clear on physical grounds that the solution we have obtained must differ from the correct solution in the region of small n/n_0 values. The reason for this can be found by examining the value of $d\eta/dr$ (i.e. the electrostatic field) which was done using the most accurate of the numerical calculations. This shows (as do the less accurate calculations) that the electrostatic field rises sharply at small values of n/n_0 and would tend to infinity as n approaches zero. In this case the assumption $n_e = n_p$ breaks down; an examination of the original equations shows that the solutions would be markedly changed when n_e and n_p differ appreciably.

The numerical calculation for this case proved to be so complicated that it has not been thought worth while to show that the solution would yield values of $n_e(r)$, $n_p(r)$ which are always positive. There is, however, no reason to suppose that one would not obtain a tailing off of the n_e, n_p curves in the region $r = 0{\cdot}25$ cm. in Figure 5, in a manner similar to that found for $\lambda = 0$.

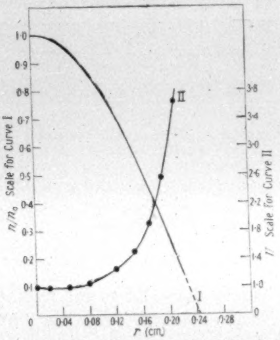

Figure 5. The radial variation of the electronic density (I) and $U = (-u_x)/D_e\xi$ (II) when $\lambda \neq 0$. $\gamma n_0 = -39{\cdot}0$, $\xi = 1{\cdot}125$, $B = 43{\cdot}2$, $A = 0{\cdot}074$.

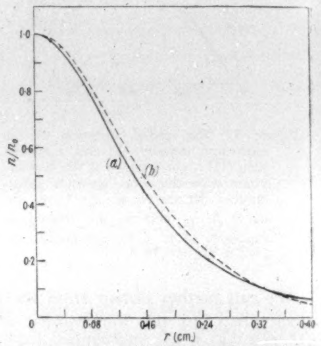

Figure 6. Comparison of the radial density variation with that for the function $n = n_0(1+\alpha_0 r^2)^{-2}$. Curve a refers to this function. In curve b $\xi = 2{\cdot}0$, $\gamma n_0 = -39{\cdot}0$, $\lambda = 0$; γ_a and γ_b are treated as functions of h.

The $h(r)$ curve shows a normal character, i.e. it rises linearly at first and then more slowly, though a maximum value is not reached at the largest value of r used.

From this curve one can obtain the longitudinal velocity in the form $-u_x/D_e\xi$. This differs from the previous set of such curves (see Figure 4) in that the values rise sharply with r (Figure 5, curve II). Though the occurrence of the extreme values are related to the effect discussed above, a significant part of the rise occurs in a region in which equations (8) and (12) should apply.

§ 5. COMPARISON WITH OTHER CALCULATIONS

A solution of the problem with which we are concerned has been given (for $\lambda = 0$) by Bennett (1934), on the assumption that the radial distribution function can be deduced from the equilibrium state. This treatment, which is relativistic, leads to the density function

$$n(r) = n_0(1+\alpha_0 r^2)^{-2}. \qquad \text{......(15)}$$

The same formula has been obtained in a recent paper by Thonemann and Cowhig (1951), using a more physical approach. Their fundamental equation for $n(r)$ is obtained by equating the diffusion 'pressure' to that produced by the electrostatic and magnetic fields. The problem is again treated as an equilibrium case, with $\lambda = 0$, and the longitudinal equation (3) of Morse and Allis is replaced by the assumption that there is a fixed longitudinal velocity independent of the radial coordinate r. The constant α_0 is expressed in terms of the total current, the longitudinal velocity, the electronic temperature and the radius of the tube. It is further shown that there is good agreement between theory and experiment.

Formula (15) has been compared in a particular case with the radial distribution calculated from the differential equations given above. The case chosen is that of $\xi = 2$, $\gamma n_0 = -39{\cdot}0$, for which the more accurate numerical integrations were performed (see § 3). The value of α_0 was fitted at small values of r. Here (15) becomes $n = n_0(1 - 2\alpha_0 r^2)$ which is identical with (13) if $b = 2\alpha_0$. The agreement obtained can be seen in Figure 6; it is reasonably good, the extreme difference being 13%. Further, it has been pointed out (§ 3), that the longitudinal velocity is nearly independent of r, the maximum deviation being of the order of 10% (see Figure 4 (*c*)).

A further paper which deals with self-magnetic fields was published recently by Schlüter (1951), after the work reported here had been completed. This differs from the present work mainly in that the initial equations do not take into account the influence of the fields themselves on the velocity distribution. These initial equations are hence simpler. With the same condition as above of zero radial current, the radial variation of the density is described by a single differential equation for $\lambda \neq 0$. A boundary condition is imposed on this equation, that the density of electrons should be zero at $r = R$, where R is taken to be larger than the radius of the tube. The $n(r)$ distribution is obtained numerically. The above boundary condition is seen to hold automatically in the $\lambda \neq 0$ case (see § 4) where the $n(r)$ curves always tend to cut the axis. As far as can be judged from the published curves, the $n(r)$ distributions seem to be similar to those given above (Figure 5, curve II).

ACKNOWLEDGMENTS

The problem with which this paper is concerned was suggested by Sir George Thomson, to whom I am indebted for helpful discussions. I should like to express my thanks to Mr. Skwirzinski for carrying out some of the numerical integrations. I am also grateful to Dr. P. C. Thonemann for drawing my attention to the papers of Bennett and of Schlüter.

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Space Current Changes in Thermionic Valves following Small Pulses of Current

By J. R. TILLMAN
Post Office Research Station, Dollis Hill, London N.W.2

MS. received 15*th May* 1951

ABSTRACT. A train of regularly spaced rectangular pulses of voltage, amplitude-modulated at a frequency f_m and applied to the control grid of a commercial valve working conventionally, caused space current changes of up to 0·01 amp/cm^2 from the oxide-coated cathode. An unmodulated pulse train, interlacing the first with a delay of t_d, sampled the anode current; the samples contained a component at the frequency f_m of low level. The component decayed as t_d increased, with a time constant of about 1 msec. It was sometimes in phase with the component obtained when $t_d=0$, and sometimes in antiphase. The magnitude and phase were often dependent on the history of the valve. Thus an operation known to poison the cathode was usually followed, temporarily, by a large antiphase component.

An in-phase component denotes an enhancement of emission following a positive pulse; an antiphase component denotes fatigue. There is evidence that the impedance at the interface between cathode core and coating is not responsible for the effects. The fatigue may well be largely a surface, rather than a bulk or matrix, effect. It bears some relationship to effects reported elsewhere with pulses involving current densities of 10 amp/cm^2.

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§1. INTRODUCTION

Coomes (1946) has reported that Ramsay of Bartol has observed two types of decay in the pulsed emission of commercial oxide-coated cathodes. The first had a time constant of the order of ten microseconds but was rarely found in very active cathodes; the second had a time constant of the order of milliseconds and was, presumably, found more generally. More recently other workers have noted similar effects. Wright (1949) for instance has emphasized the disparity which exists between the pulsed emission of oxide-coated cathodes and the D.C. (continuous) emission; he attributed the disparity to a decay effect. He found the more rapid of Ramsay's decay to be rare in his commercial cathodes, but the second to be almost always present. He found no time dependent effects at 'low' current densities ('low' would seem to mean less than 100 ma/cm^2); he was in general, however, observing differences of currents of up to ten to one and might well not have concerned himself with changes less than, say, 1%.

Feaster (1949), after having noted that many valves do not show a rapid decay of emission during pulses of duration of about 10 μsec., has described how valves with anodes coated with sodium chloride show much decay during these pulses when the current density at the cathode is about 5 amp/cm^2.

Very recently, Forman and Rouse (1951) have studied the effects caused by the presence of small quantities of magnesium in the nickel of cathode sleeves; they noted, when pulsing carefully made diodes at current densities of the order of 1 amp/cm^2, that the current passed fell appreciably during pulses of duration 250 μsec.

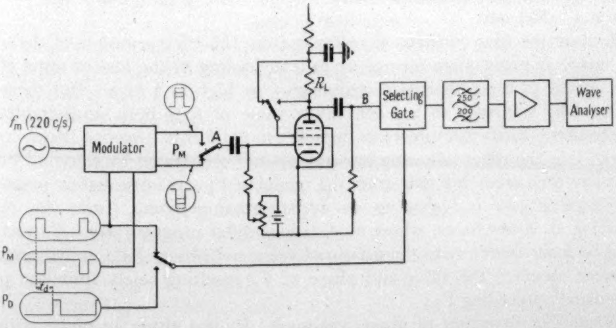
The effect of pulsing on the space current of a valve now reported arose when Flood and Tillman (1951) attempted to confirm, experimentally, new analyses of the interchannel interference (crosstalk) which arises, in multichannel systems of telephony using time division and amplitude modulation, from the deficiencies of the common transmission path. Early measurements showed an additional source of crosstalk which was traced to a change of, perhaps, 0·1% in the space current, in a valve in the path, following a pulse of additional current. The valve had a conventional indirectly heated, oxide cathode. The space current, except during pulsing, was never more than 10 ma. (corresponding to 10–20 ma/cm^2) and the pulse never more than 10 ma. The change of current, though small, bore some resemblance to those reported by Coomes, Wright, etc. The original apparatus was first modified to minimize the effect, but was later adapted to measure it.

The magnitude of the effect could cause some concern in the design of telephone systems using time division and amplitude modulation. Thus it might well be significant in a ten-channel system with a pulse repetition frequency of 8 kc/s. (channel spacing 12·5 μsec.) and a duration of channel pulse of, say, 8 μsec. Designers of good quality systems might well have to find ways of avoiding it. The mere collection of data, over a wide range of small valves, is, however, probably not worth while. On the other hand, the effect may be of importance in the physics of the oxide-coated cathode, particularly as it can be partially correlated with the treatment and operation of the valve and the better known properties of the cathode.

§ 2. EXPERIMENTAL

2.1. *General*

In the apparatus used to measure the effect, shown in the Figure, the electrodes of the valve under test, e.g. a pentode type CV138 (6F12) or CV1065 (SP61) received conventional voltages. The potential of the first grid V_{g1} was adjusted



to give a space current I_k of 8 ma.; of this current about 2 ma. flowed to the screen grid G2 and about 6 ma. ($=I_{a0}$) to the anode A, whose circuit contained a resistive load, R_L. G1 received a pulse of voltage every 1,000 μsec., whose unmodulated amplitude v was about 0·5 v. and whose duration was 100 μsec. (or, for a few experiments, some shorter period). The pulse could be of either polarity.

It would have proved very difficult to measure a small difference between the space current immediately before a pulse and that at some time later merely by periodically sampling the two currents and comparing them after D.C. amplification. Instead, the train of pulses P_M was modulated by a sinusoidal signal, of frequency f_m (usually about 220 c/s.), whose magnitude was adjusted to give a depth of modulation m of about 80%. Thus the anode current during a pulse was $I_{a0} \pm gv\{1+m\cos(2\pi f_m t+\phi)\}$ according to the polarity of the pulse, where g is the mutual conductance of the valve (usually about 5 ma/v.) and ϕ is a constant. A selecting gate pulse P_D also occurring every 1,000 μsec. had a duration adjustable from 10–100 μsec.; its leading edge could be delayed with respect to that of P_M by any time t_d up to 900 μsec. It sampled the voltage across R_L as a measure of the anode current I_a.

When $t_d=0$, the voltage after sampling was $R_L[I_{a0} \pm gv\{1+m\cos(2\pi f_m t+\phi)\}]$ at times common to pulses of both trains P_M and P_D, $R_L I_{a0}$ for any time by which a pulse P_D was prolonged after the end of a pulse P_M, and zero otherwise. The waveform of this voltage is known to contain a component of frequency f_m, which after amplification was measured by the wave analyser as a voltage V_0. When $t_d > 100\,\mu$sec., however, the pulse trains P_D and P_M never overlapped, so that the voltage sampled was, nominally, $R_L I_{a0}$ and no component V_d of frequency f_m was expected to appear at the input to the wave analyser. Although the inevitable restriction of the bandwidth of the transmission path from A to B introduced crosstalk (i.e. gave V_d a finite value), component values were so chosen that the crosstalk attenuation V_0/V_d was at least 90 db. (30,000 times) for all values of t_d greater than 120 μsec.; crosstalk other than that introduced by the transmission path made an even smaller contribution to any residual value of V_d.

None the less, when t_d was, say, 200 μsec., V_d/V_0 was usually 0·001 to 0·0001, corresponding to an attenuation of only 60–80 db.; it far exceeded any expected residuals. The attenuation increased with increase of t_d, being a few decibels more at $t_d = 500\,\mu$sec.

Because the time constant of recovery from the effect noted is of the order of 1 msec., it might seem unwise, or at least leading to the loss of some of the finer points, to use a repetition frequency f_r as high as 1 kc/s. But there are reasons why a change to a much lower value of f_r is both undesirable and unnecessary. First, measurements become more difficult. Second, experiments showed that the effect following one pulse is not obliterated by a second pulse; V_d at any time arises not only from the modulated pulse immediately preceding the selecting gate but also, to an ever-decreasing extent, from the pulses occurring at earlier times, whose modulations differ progressively. Corrections could be made therefore to the measured value and phase of V_d, to deduce with sufficient accuracy the value and phase of V_d resulting solely from the pulse immediately preceding P_D.

When the correction to phase was made, V_d was either in phase with V_0 (and hence V_d/V_0 could be spoken of as positive) or in antiphase with V_0 (when V_d/V_0 could be spoken of as negative).

When P_M was a train of positive-going pulses, V_d/V_0 was, in general, greater if it was negative and smaller if it was positive than when the train was negative-going.

Triode-connection of the valve gave results usually only slightly different from pentode-connection. A few experiments showed V_d/V_0 to be substantially independent of the depth of modulation and not a rapid function of the unmodulated pulse height. It was, however, often dependent on the quiescent cathode current I_k; V_d/V_0, if positive, increased with increase of I_k, but if negative behaved in no consistent way from valve to valve, sometimes becoming more negative with increase of I_k, sometimes even becoming less negative or positive.

As described in a later section, V_d was often affected by treatment of the valve, through temporary change of operating conditions.

2.2. *Results for Valves before any Special Conditioning*

V_d/V_0 at $t_d = 200\,\mu$sec. was only occasionally less than 0·0001 for new valves. For the type CV 1065 and for valves made in the Thermionics Group of this establishment to the specification of the CV 138, V_d was almost always in phase with the modulation on the pulse of current passed (i.e. V_d/V_0 was positive) when the heater voltage V_h was the rated value (6·3 v.); for a minority of them V_d became in antiphase (i.e. V_d/V_0 became negative) when V_h was reduced to 4·0 v. or less. The reversal of phase could lead to values of V_d/V_0 as small as 0·00003 at some intermediate heater voltage. Most of the commercial valves type CV 138 tested gave negative values of V_d/V_0, irrespective of V_h.

Some valves of the type CV 1065 which had already run for 60,000 hours were available for test. They retained an emission comparable with that of new samples, but the interface between the coatings and the sleeves of their cathodes had developed impedances of some tens of ohms, whereas samples from current production had impedances of only an ohm or two. These much aged valves gave however amongst the smallest values of V_d/V_0 noted.

2.3. *The Effects of Short Term Conventional and Unconventional Usages of the Valves*

The more important conditions applied to the valves from time to time were:

A with $V_a = V_{g2} = 150$ v. and $I_k = 5$ ma., often for many hours or days.

B as diodes with $I_k = 5$ ma., often for many hours or days.

C as B but $I_k = 25$ ma.

D for 30 sec. with $V_a = 0$, $V_{g2} = 200$ v., $I_{g2} = 20$ ma., thereby much exceeding the maximum rated dissipation of G2.

E for 30 sec. with $V_a = V_{g2} = 200$ v., $I_a \simeq 16$ ma. and $I_{g2} \simeq 4$ ma.

F with $V_a = V_{g2} = 0$, $V_h = 4{\cdot}0$ v., $V_{g1} = 7$ v. for 5 minutes. The initial cathode current p fell to a final current q.

G as F, but with $V_h = 7{\cdot}0$ v. and lasting 1 hour.

H with $V_a = V_{g2} = 50$ v. and $I_k = 25$ ma., for 10 minutes.

J as F but $V_{g1} = +13$ v.

K with $V_a = V_{g1} = V_{g2} = -9$ v., for many hours.

L as J but for 1 hour.

Condition A is typical of everyday use and does not normally cause rapid changes in the state of the cathode coating. It rarely affected V_d, unless some previous treatment, e.g. F, had seriously changed V_d, when A would usually restore V_d to a more stable value, often that of the valve when new.

Conditions B and C, though not typical of everyday use, would hardly be expected to cause rapid damage to the cathode coating; indeed because they involve no positive ion bombardment of the cathode, they may well undo some of the slow damage resulting from prolonged, more conventional, use. Both conditions had an influence on V_d, even if they followed a long period of A, C much more so than B. Negative values of V_d/V_0 were either much reduced or were changed to small positive values. Positive values were less affected; some were, however, significantly reduced.

Operation D often caused temporary softness in the valves, particularly those of type CV 1065, and usually some change in V_d. The softness gradually disappeared within a minute or so, and with it most of any change in V_d. When, before D, V_d/V_0 was positive, D usually reduced it and occasionally reversed its sense; when, before D, V_d/V_0 was negative, D often increased it.

Operation E is less severe on the valve than D; it caused only small changes in V_d.

Operation F is thought (Metson 1949) to be able to dissociate those compounds present on the control grid whose heats of formation are less than 7 ev. The dissociation gives rise to gas which finds its way to the cathode; usually the emitting surface is poisoned and q is less than p. The emission, if so reduced, can usually be restored substantially to that applying before F, by running the valve for a minute or so with conventional voltages applied to its electrodes. When q was less than $p/4$—and for some valves it was as little as $p/20$—V_d/V_0 was usually much changed, for a few seconds at least; it was then always negative and often large, e.g. 0·003. When q was greater than $p/2$—and for some valves it was as much as $0{\cdot}9p$—positive values of V_d/V_0 were usually reduced but negative values changed little. If V_d/V_0 had become large and negative following F, its recovery to a more normal value could be retarded by reducing the voltage of the supply to the heater of the valve when it was in the apparatus measuring V_d/V_0.

Operation G carries operation F to greater lengths as far as ridding the control grid of those compounds mentioned above is concerned. But the higher heater voltage applied throughout ensures that recovery, or reactivation, of the cathode takes place so rapidly that the reduction of emission resulting from the poisoning, which is also going on, is only just measurable. G sometimes affected V_d/V_0, driving it negative or more negative. It much reduced the effects of any further operations F.

Operation H had little or no effect on V_d/V_0.

Operation J resembles F but can dissociate some compounds unaffected by F. It affected V_d/V_0 in the same sense as did F, but usually much less so.

Condition K suppresses all space current. It has been said to be capable of accelerating valve ageing, possibly by facilitating the growth of a poorly conducting interface between the coating and the sleeve of the cathode. A few hours or days of running under condition K had little effect on V_d/V_0. Time did not permit of longer periods, although several months if not years of condition K are normally necessary to promote a substantial interface.

Operation L was to J much as G was to F. It never resulted in any important changes to V_d/V_0.

The effects of H, J, K and L were not investigated for many valves.

§ 3. DISCUSSION

Because the effect noted could be either a value of V_d in phase with V_0 or a value in antiphase, it ought perhaps to be looked upon as two effects, both being usually present, but one exceeding the other. The argument for simultaneity is strengthened by the results for some valves for which V_d/V_0 was positive though small for values of t_d less than about $200\,\mu$sec., and negative though small for greater values of t_d, or vice versa.

Discussion is simplified if it is confined to the effects following positive pulses of cathode current; it enables a direct comparison to be made with the results of the workers using pulses of high current densities. Everything said will, however, have its counterpart for negative pulses.

A positive value of V_d/V_0 can reasonably be called an enhancement of emission and a negative value a fatigue.

There is clearly an important correlation between the effects and the state of the cathode; the enhancement was most significant when the cathode had been so treated that poisoning had been avoided and fatigue most significant when poisoning had been intense.

Although the effects have time constants of recovery to normal of the same order as those of decay observed by Ramsay, etc., there are at least two reasons why they must not be assumed necessarily to have the same origins as those resulting from pulsing with current densities of 1 amp/cm^2 or more. First, the experiments with high densities of current deal almost entirely with fatigue. Second, the effects now reported seem to be as much a function of the state of the cathode before the pulse, brought about by previous treatment, as of any change of state effected by the pulse.

The causes of the effects cannot here be more than the subjects of brief speculations. They can hardly arise from the mere presence of an interface between the sleeve and the coating of the cathode. The time constant is several powers of ten removed from that of interface impedances; moreover the aged valves, with much increased impedances, showed no more effect than did new valves with very small impedances.

The causes may lie with the body of the oxide layer, e.g. in changes of conductivity. There are, however, such conflicting views as to the mechanism of conduction (Loosjes and Vink 1949, Hannay, McNain and White 1949) and doubts about the importance of electrolysis and electrolytic conduction (Sproull 1945, Wright 1950) in controlling emissivity that speculation is dangerous. Because increased activation of the cathode can sometimes be achieved by drawing a large current through it, the explanation of the enhancement found may merely be that of this means of activation. The enhancement cannot, however, be due to temperature changes during the pulse, which are of the wrong sign when the heat lost (or gained) due to the change of emission during a positive (or negative) pulse is taken into account.

Thus in the experiments described the change of dissipation produces about 10^{-8} cal. per positive pulse in the oxide layer if its resistance is as much as 5 ohms, and about 10^{-7} cal. are required to account for the increased space current (assuming a work function of 2 volts). Because the thermal capacity of the oxide layer, whose thickness is about 10^{-2} cm., can hardly be less than 10^{-3} cal/° c., the mean temperature change is hardly likely to exceed 10^{-4}° c. On the other hand the relationship between space current and cathode temperature was, under the conditions of test, about 0·02 ma/°c., so that the effects noted—changes of current of 0·0004 ma. or more—would have demanded a temperature change of at least 10^{-2} °c.

It is tempting to explain the fatigue primarily as a surface effect; if the surface has been poisoned, each pulse, if positive, tends to exhaust the cathode, so that when it has passed, the emission preceding it is not immediately forthcoming; a negative pulse on the other hand rests the cathode momentarily, enabling it to pass more current. The experiments with operation F afford the most striking illustration for this explanation. Wright favours surface effects as the cause of fatigue during pulses of high current density.

The electrical effects noted are perhaps merely further manifestations of the continuous metamorphosis of the oxide-coated layer. They may, however, be of use in understanding the operation of the layer if only in conjunction with other experiments. They may also be of use in determining the state of the cathode; if they can be shown to be of use in predicting the useful life of a valve, they may have an advantage over the measurement of total emission, which must necessarily involve either a cathode temperature or a space current very different from that of normal usage.

§4. CONCLUSIONS

The space currents before and after the passage of a positive or negative pulse of current of a few milliamperes differ. For some valves the change has the same polarity as the pulse, when it can be called an enhancement; otherwise it has the opposite polarity and is called a fatigue. The history of the valve's usage often determines the polarity and magnitude of the difference. Two causes seem necessary to explain the results. It seems hardly wise at this stage to link the causes closely with those postulated by workers observing decay of emission during pulses of very high current density, although the time constant of recovery from the effects noted is much the same as those of the decays reported elsewhere.

ACKNOWLEDGMENTS

The author wishes to thank Dr. L. E. Ryall for many helpful discussions, Mr. R. E. Warren for making many measurements, and the Engineer-in-Chief of the General Post Office for permission to make use of the information contained in the paper.

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Ionospheric Interaction in Disturbed Conditions

By D. A. BELL*

Department of Electrical Engineering, University of Birmingham

MS. received 29th March 1950, *and in amended form 25th June* 1951

ABSTRACT. Measurements of ionospheric interaction depend on special radio transmissions which are only available on a limited number of occasions, and only on some of these occasions are ionospheric conditions steady enough for direct comparison between experiment and theory. This paper describes the use of averaging methods to extract some information from observations made when conditions are not steady. The results confirm the existence of a 'low-frequency anomaly' observed by Huxley, but show no exceptional interaction for a variation of the frequency of the disturbing wave around the nominal gyromagnetic resonance frequency.

§ 1. INTRODUCTION

The theoretical analysis of Bailey (1937 a, 1938) has been confirmed by observations at Birmingham (Huxley *et al.* 1948, Huxley 1950) and at Cambridge (Ratcliffe 1948) in the sense that during favourable conditions of observation the relation between transferred modulation and frequency of modulation is of the correct form in magnitude and phase; but there are a number of secondary points which need investigating. Huxley and Ratcliffe reported that no substantial correlation between the interaction and the direction of propagation relative to the earth's magnetic field had been found, and the experimental evidence on the existence of a gyromagnetic resonance effect is so varied as to appear inconclusive. It ranges from the qualitative positive result reported by Bailey (1937 b), based partly on comparison between the effects of transmitters so far separated in frequency (1,195 kc/s. and 877 kc/s.) that they might have been effective at different heights in the ionosphere, through the results of Cutolo *et al.* (1946), who found extraordinarily strong interaction for quite low transmitter power, to the completely negative observation at Birmingham in 1949 (see below) in which, however, the frequency variation may have been too small to sweep through the resonance curve.* The second point is that in some conditions the intensity of transferred modulation is less at a modulation frequency of 60 c/s. than at rather higher frequencies, which is contrary to the theoretical analysis as usually presented.

The objects of the present paper are therefore as follows: (*a*) to suggest methods of discriminating between random fluctuations in the observations and significant trends, on occasions when the results would otherwise have been discarded on account of excessive variability; (*b*) to assemble sufficient evidence of the existence of the 'low-frequency anomaly' (the second point above); (*c*) to record briefly the negative result of the Birmingham observations on a gyro-resonance test on 8th July 1949; (*d*) to suggest the type of modification to the theory which might lead to an explanation of the low-frequency anomaly.

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* Huxley and Ratcliffe (1950) disagree with Bailey's estimate of the magnitude of effect to be expected near the gyrofrequency.

§2. INTERPRETATION OF SCATTERED OBSERVATIONS

In all but the most favourable conditions, the depth of transferred modulation at a given modulation frequency will vary from minute to minute, in a way similar to the general fading of intensity of a signal transmitted via the ionosphere. ('Depth' of transferred modulation here refers to percentage modulation, so that the variation in question is additional to and separate from fading of the carrier.)

The observations reported in this note were made on transmissions in which the modulation frequency was changed at half-minute intervals, so that several minutes were required to cover all frequencies; and the random variations could therefore be comparable with the systematic effect of change of modulation frequency. In spite of this, and other difficulties*, the author considered that the rarity of special test transmissions for ionospheric investigation justified an attempt to extract some information from a necessarily imperfect experiment. On the nights in question the random variations were considerable, so that it is necessary to examine the dispersion of the readings and attempt to find an average value.

The simplest method is to take a set of observations which appear homogeneous, arrange them in order of magnitude, and plot them at uniform percentage intervals on probability graph paper: if the distribution is gaussian, this plot will be a straight line. Figure 1 shows a plot of this kind of a group of readings of transfer of 120 c/s. modulation, the group appearing reasonably homogeneous internally and being obtained during a period when the mean carrier level was fairly constant. From curve A, of the points plotted as percentages, it is clear that the lowest four points are not homogeneous with the main group, and that the distribution of the main group is not gaussian.

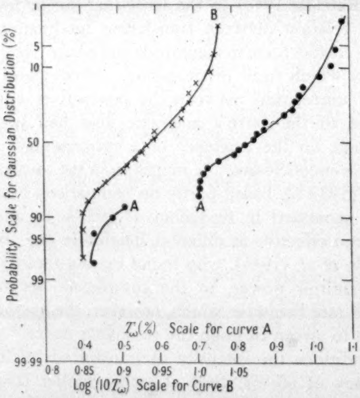

Figure 1. Distribution of observations for a single modulation frequency (120 c/s.), showing the proportion of observations which would be expected to fall above a given value of transferred modulation T if the distribution were gaussian. Curve A: distribution of percentage transferred modulation. Curve B: distribution of logarithm of percentage transferred modulation.

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* Most of the original team of experienced observers had left Birmingham.

Owing to successful use of decibel units elsewhere, it was decided to re-plot the main group of these observations on a logarithmic scale of depth of modulation. This has some theoretical support, since the transfer of modulation is related to an absorption effect which varies exponentially with length of path and absorption coefficient, and therefore a random variation of length and absorptivity of the path through the interaction region should give a random variation in the logarithmic measure of depth of transferred modulation. The central part of this curve (B in Figure 1) is perhaps more linear than the corresponding part of A, but both very low and very high values are less frequent than they would be in a gaussian distribution. The shortage of very low values may be due to the background of noise and interference, but the scarcity of very high values appears to be inherent in the distribution. But curve B does suggest a stable distribution, from which one could reasonably take an average value for comparison with the averages of other groups of observations. The three different means which one could take have values as follows: (*a*) arithmetic mean of all observations plotted on curve B, 0·875%; (*b*) mean of logarithms plotted, 0·867%; (*c*) 50% mark on curve B, which falls at a value corresponding to 0·871%. The differences between the three means are small, and if the distribution were gaussian throughout, the standard deviation would be about ±0·05%, which is greater than the difference between (*a*), (*b*) and (*c*); the arithmetic mean is therefore sufficiently true. Curve B suggests that if one can collect about twenty observations which relate to similar conditions, the mean should be reliable to better than ±1 part in 10, when the range of individual observations is of the order of ±40% of the mean. Figure 2 shows a set of (T_ω, ω) curves

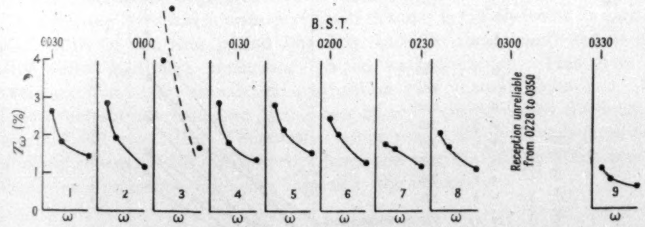

Figure 2. Set of average curves for each ¼-hour period of percentage transferred modulation plotted against modulation frequency at values 60, 120 and 300 c/s.

(T_ω = transferred modulation at modulation frequency $\omega/2\pi$) for different quarter-hour periods during the night. Three modulation frequencies were available, 60, 120 and 300 c/s., and each plotted point in Figure 2 for a particular modulation frequency and quarter-hour period represents the average of from 2 to 45 readings (one reading only for the very high point 3). The significance of the varying shapes of these short arcs of the full (T_ω, ω) curves can be appreciated by comparing them with the normalized form (Figure 3) of the theoretical curve $T_\omega = T_0\,[1+(\omega/G\bar{\nu})^2]^{1/2}$ given by Huxley and Ratcliffe (1949). By comparison with Figure 3 it will be seen that a (T_ω, ω) curve which is markedly concave upwards, as the curves of groups 1 and 4 of Figure 2, corresponds to $\omega/G\bar{\nu} \simeq 2$ at 120 c/s. and T_0 large, while a curve such as group 7 corresponds to $\omega/G\bar{\nu} \simeq 0{\cdot}5$ at 120 c/s. and T_0 not so much greater than the transferred modulation at 60 c/s.

If $1/T_\omega^2$ is plotted against ω^2 the theoretical formula is a straight line which can be extrapolated back to cut the ω^2 axis at a negative value equal to the magnitude of $(G\bar{v})^2$ (Huxley *et al.* 1947); and the data of Figure 2 have been plotted in this manner in Figure 4. Only in the case of group 7 do the three points fall on a straight line; in all the other groups the 60 c/s. point lies below the line through the 120 and 300 c/s. points. In these cases there is no unique line on the basis of internal evidence, but external evidence provides two reasons for drawing the line through the 120 and 300 c/s. points and ignoring the 60 c/s. point: (*a*) this yields values of $G\bar{v}$ comparable with that given by the one consistent group (7) and with the values reported elsewhere for observation under more favourable conditions, and (*b*) although the present case, with three

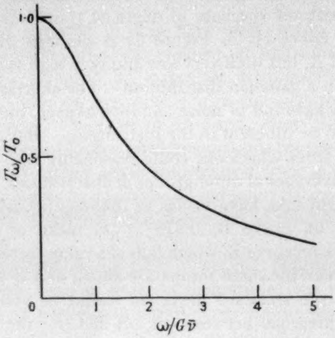

Figure 3. Normalized curve for variation of transferred modulation (relative to value T_0 for zero frequency) with ratio of angular frequency ω of modulation to collision parameter $G\bar{v}$.

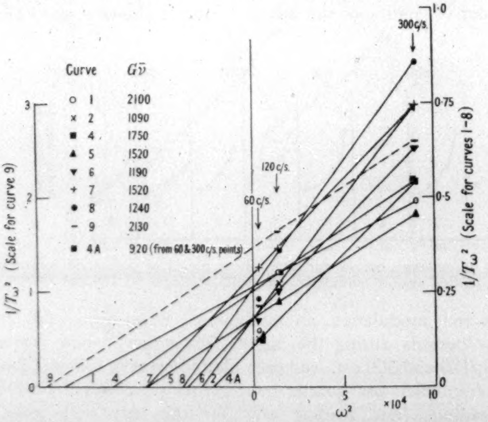

Figure 4. Extrapolation of relation between $1/T_\omega^2$ and ω^2 to determine $G\bar{v}$.

frequencies only, does not indicate which is wrong, observations on other occasions have revealed a tendency for the points at higher modulation frequencies to lie on a straight line with that at 60 c/s. off the line (see § 3 below). A value of $G\bar{\nu}$ for each group has therefore been calculated from the 120 and 300 c/s. readings, and these values are plotted in Figure 5 against time for comparison with the simultaneous figures of average carrier level.

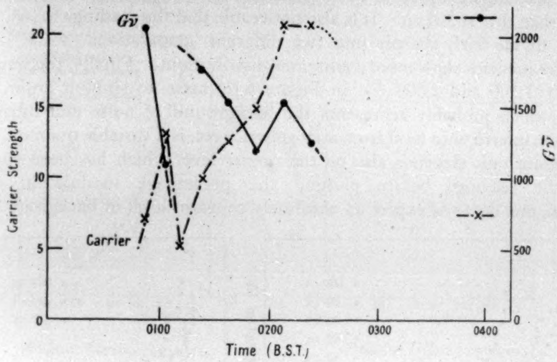

Figure 5. Comparative plot of carrier strength and estimated value of $G\bar{\nu}$ against time.

The mechanism suggested by the data in Figures 2 and 5 is as follows: The reflecting layer which was operative at the start is rapidly losing its ionization ($G\bar{\nu}=2{,}100$ in the first period but only 1,030 in the second) and at about 0108 hours is almost penetrated by the wanted carrier. This gives high attenuation and large transferred modulation with unsteady behaviour. After 0115 penetration is complete, and a new layer is operative with $G\bar{\nu}$ fluctuating around 1,500, until 0230. There is then a long period of very disturbed conditions until we return to the initial carrier level and to $G\bar{\nu}=2{,}120$ at 0350, probably due to new ionization following sunrise in the ionosphere.

§ 3. THE 'LOW-FREQUENCY ANOMALY'

The scatter in the readings with step-by-step change of modulation frequency is almost always so great that one can select a number, perhaps a majority, of observations which fit the theoretical curve, while there are others which depart widely from it. In such cases the evidence for a departure from the theoretical curve must be mainly statistical, in contrast to one experiment (Huxley 1950) which established absolutely the existence of the low-frequency anomaly in qualitative form, when using simultaneous modulation of the interfering carrier by two frequencies.*

On the lines described in § 2, the whole of the observations for the period 0120 to 0415 B.S.T. on the morning of 27th July 1949 were plotted as distributions on probability graph paper. (The 'wanted carrier' was radiated by Lisnagarvey on 1,050 kc/s. and the modulated disturbing signal from

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* Huxley's report of this experiment was not available to the author when these experiments were carried out and analysed.

Droitwich on 200 kc/s. with reception at Birmingham. The modulation frequencies used were 60, 120, 300, 700, 1,000, 1,250, 1,500 and 2,000 c/s. during successive half-minute intervals, and the modulation-frequency output from the receiver was measured with a wave analyser having a band width of approximately 4 c/s.). The observations relating to modulation frequencies of 60, 120, 300, 700 and 1,250 c/s. are reproduced in Figure 6 (*a*), where it is obvious that the transferred modulation at 60 c/s. is consistently below that at 120 c/s., and not greater than that at 300 c/s. It is also noticeable that the readings for 60, 120 and 300 c/s. divide fairly sharply into two different 'populations' while those for higher frequencies show more continuous distributions. Finally, the frequencies of 1,250, 1,500 and 2,000 c/s. in Figure 6 (*b*) seem to show a lower limit of 0·05%, which probably represents the background of noise and interference. (Since the interference level from atmospherics etc. is a variable quantity, and the signal–noise ratio depends also on the *carrier* level which has been eliminated from the readings before plotting the percentage modulation data in Figure 6, one does not expect an absolutely constant level of background: hence a few points fall below the average cut-off level of 0·05%.) For these modulation frequencies the best mean value of T_ω is that taken from Figure 6 as the value where the line through the points above 0·05% modulation (extrapolated if necessary) cuts the line marked 50% on the probability scale.

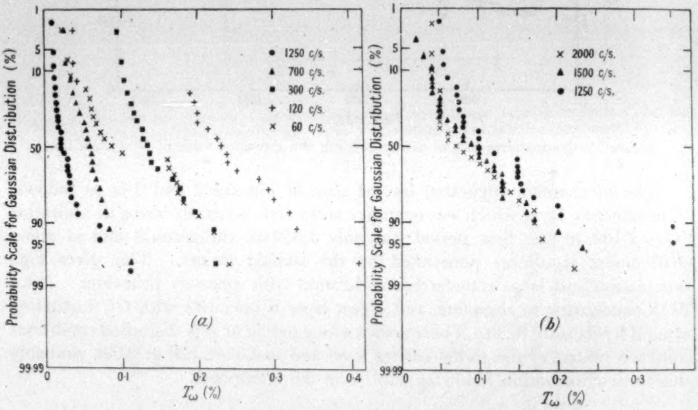

Figure 6. Distribution of percentage transferred modulation.

In a linear plot of $1/T_\omega^2$ against ω^2 undue weight is given to readings with large values of ω and small (and therefore less certain) values of T_ω, and a logarithmic scale has therefore been used, with no attempt to adjust reference points, since a preliminary linear plot showed that when the low-frequency anomaly was present, a large part of the data (i.e. readings for the higher values of ω) approximated to a straight line which differs from the theoretical one, having an intercept on the ω axis small and apparently *positive*. The averaged data (Figure 7) are in the following form: the averages for 'all' the experimental observations, plus the average for the 'high' distribution and the average for the 'low' distribution for those cases where the total distribution, as plotted in Figure 6, appears to include two separate groups of points. Figure 7 shows that, on any interpretation of the data, the interaction effect is less at a frequency of 60 c/s. than at 120 c/s.

It was at first suspected that the low-frequency anomaly might be due to some instrumental error (Huxley's double-tone results were not known at the time), and confirmation was sought from other receiving stations. Figure 8 is plotted from data supplied by the B.B.C. Research Department* of reception from Tatsfield on 18th May 1949, 0035–0350 B.S.T. The wanted carrier was Lisnagarvey, 1,050 kc/s., and the disturbing signal Droitwich, 200 kc/s. Individual readings were recorded in decibels relative to 5% modulation, and for any one modulation frequency show a spread of about 6 db.; averaging and conversion to percentage modulation gave the smooth curve Figure 8. This confirms both the reduced interaction at low frequencies and the incorrect slope at higher frequencies which was deduced from Figure 7.

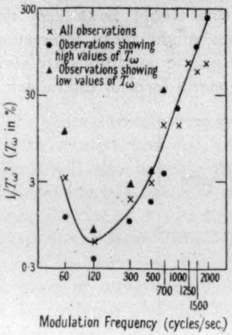

Figure 7.

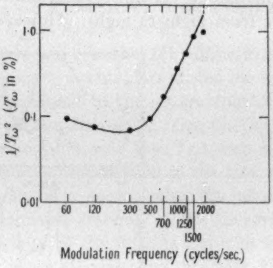

Figure 8. The low-frequency anomaly deduced from data supplied by the B.B.C. receiving station.

As far as can be gathered from the small number of tests at present analysed, the conditions which lead to a low-frequency anomaly in the (T_ω, ω) curve include (*a*) summer night conditions in the ionosphere and (*b*) low-frequency disturbing signal acting on a medium-frequency wanted carrier. A theoretical consideration of the anomaly is given in § 5.

§ 4. GYRO-RESONANCE TESTS 6TH and 8TH JULY 1949

In the gyro-resonance tests of 6th and 8th July 1949 the wanted carrier was radiated from Westerglen on 767 kc/s., and the disturbing carrier from Ottringham, its frequency being varied in 5 kc/s. steps at quarter-hour intervals from 1,325 kc/s. to 1,355 kc/s. and back again. The intensity of interaction certainly varied considerably during the night, but behaved similarly on 27th July when the stations used were Lisnagarvey wanted (1,050 kc/s.) and Droitwich disturbing (200 kc/s.); these results indicate that the variations are due to changes in the ionosphere rather than to changes of carrier frequency. Intensity of 60 c/s. transferred modulation, relative carrier level of wanted signal, and frequency of disturbing signal are plotted against time in Figure 9.

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* The original readings were lent to the author by the B.B.C., and it must be emphasized that the averaging and interpretation is the responsibility of the author only.

There is some doubt whether a frequency swing of ±15 kc/s. is sufficient to include an appreciable part of the gyro-resonance curve if there is in fact a significant effect of such resonance on the relationship between interaction intensity and carrier frequency. One might then consider whether the average intensity of interaction observed with the 1,340 kc/s. disturbing signal was much greater than with other frequencies. But this would be a very doubtful comparison since the use of other frequencies might mean interaction in a different part of the ionosphere, and the ionospheric conditions are also liable to vary from night to night. However, the general level of interaction shown in Figure 9, 1–3%, is comparable with that observed on other occasions (Huxley *et al.* 1948). It is therefore concluded that these observations show no positive evidence of enhanced interaction near the gyro-resonant frequency, although they do not exclude the possibility of such resonance in some conditions.

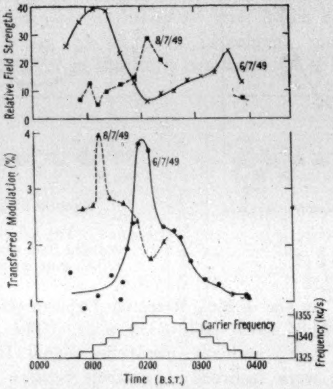

Figure 9. Variation of received carrier strength and transferred modulation during frequency variation of the interfering carrier about the gyro-resonant frequency.

§ 5. POSSIBLE MECHANISM OF LOW-FREQUENCY ANOMALY

A cause of irregularity at the high-frequency end of the (T_ω, ω) characteristic is the existence of interaction at two different regions in the ionosphere (Huxley 1950), with a phase difference of $2\pi l/\lambda$, where l is the path difference and λ the wavelength of the modulation frequency. This mechanism seems impossible for an anomaly at a frequency as low as 60 c/s., since the path difference would be excessively large.

The most likely source of a time constant of the order of 5–10 milliseconds (since the low-frequency anomaly shows maximum interaction occurring at 100–200 c/s. instead of at zero frequency) is in the rate at which the free electrons in the ionosphere absorb and lose the additional energy which is imparted to them by the disturbing wave, and it seemed to the author that there might be an analogy with the question whether a sound wave would be propagated adiabatically or isothermally in a given medium.*

The usual method of calculating the interaction (see, for example, Huxley and Ratcliffe 1949) is to use the approximation that the collision frequency can be taken as a constant $\bar{\nu}$ during the modulation cycle for the purpose of calculating the rate of loss of energy by collision (although it is in fact the variation of ν during the modulation cycle which causes interaction); write

$$\begin{gathered}(d/dt)Q(t)+G\bar{\nu}[Q(t)-Q_0]=P(t)\\ \text{rate of change of K.E.}+\text{dissipation}=\text{power absorption}\end{gathered}\qquad\ldots\ldots(1)$$

and assume that $P(t)$ is obtained simply by substituting $\mathscr{E}(t)=\bar{\mathscr{E}}(1+M\cos\omega t)$ into a steady-state formula $P_1=f(\nu, p_D)\times\mathscr{E}_D{}^2$, where p_D is the (angular) carrier frequency of the disturbing signal and $\mathscr{E}_D$ its field strength.

The general result of replacing ν by $\nu(t)$ in equation (1) will be to reduce the intensity of interaction, for, when the electron energy rises during the modulation peaks, ν will also increase (being proportional to the square root of the total agitational energy), and this will increase the dissipation term $G\nu[Q(t)-Q_0]$, so limiting the rise of Q to a lower value than it would have had with ν remaining constant at the mean value. But since ν^2 is proportional to the *total* agitational energy of the electrons, this effect can only be important when the increase due to absorption of wave energy is an appreciable fraction of the thermal energy; and the evidence provided by the linearity of the relation between transferred modulation and power of disturbing wave (Huxley and Ratcliffe 1950) gives no positive support to the idea that the wave energy is comparable with the thermal energy. Moreover, although the mean wave energy may rise slightly at the lower frequencies as it follows the modulation, the author has not been able to find any quantitative modification of the quasi-equilibrium treatment of the process which will account for an actual reduction in transferred modulation at 60 c/s. relative to 120 c/s.

It therefore seems that the 'power absorption' in equation (1) must be regarded as a function of the instantaneous disequilibrium between the field strength and the electron energy, and not merely of the equilibrium condition which would eventually be attained at a given power level, and that the modulation component of fundamental frequency, as well as the distortion, is modified by this. The general experimental evidence indicates that $\omega < G\nu$ when $\omega/2\pi=60$ c/s., so that at this frequency the rate of loss of energy from the electron should be able to keep pace with the falling part of the modulation cycle. The other factor is the rate of absorption of energy during the rising part of the cycle. In this connection it should be noted that the approximate formula for power absorption (Huxley and Ratcliffe 1950) assumes that the squared collision frequency ν^2 is small compared with $p_D{}^2$ already defined. But since $G\nu$ is of the order of 1 to 2×10^3, while estimates of G range from 1·3 to $2{\cdot}6\times10^{-3}$, it follows

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* The point was expressed in this form in a report of the observations which was circulated to the Radio Research Board in August 1949. A similar idea was included in a note (unpublished) communicated by L. G. H. Huxley to a discussion in Birmingham in November 1949, where he comments on the limitations of the formula $T_\omega=T_0[1+(\omega/G\bar{\nu})^2]^{-1/2}$: "The formula in question is obtained when the collisional frequency ν in the factor $G\nu$ is replaced by the mean value $\bar{\nu}$ and treated as a constant in this factor." This would account for observations showing the transferred modulation reduced at frequencies below 100 c/s., and "The formal rigorous solution [of the equation of energy balance of the electrons in the field] is

$$Q-Q_0=\left[\exp\left(-\int_0^t G\nu\,dt\right)\right]\times\int_0^t \omega(t)\left[\exp\left(\int_0^t G\nu\,dt\right)\right]dt+\text{const.}\times\exp\left(-\int_0^t G\nu\,dt\right)\text{''}.$$

that $\bar{\nu}$ is not much less than 10^6 and therefore comparable with the angular frequency of a long-wave transmitter (e.g. 200 kc/s. or 167 kc/s. carrier frequency).* As noted at the end of § 3, the low-frequency anomaly seems to be dependent on the use of a low-frequency disturbing signal, so that ν^2 may be comparable with p_D^2.

ACKNOWLEDGMENTS

The author is very conscious that in this work he is building on the foundations laid by others, and hopes that they will appreciate that, in so far as this paper is critical of the simple theory, the criticism is intended to be constructive. Particular thanks are due to Professor L. G. H. Huxley, whose work at Birmingham up to the end of 1948 provided both the occasion and the equipment for the author's observations in the summer of 1949, to Mr. C. C. Newton, who assisted in the experiments on several of the nights for which results have been used here, and to Mr. F. J. Hyde, who provided continuity with Professor Huxley's work. In addition, the author wishes to thank Professor A. Tustin for encouraging the work, the various members of the Electrical Engineering Department who helped to erect and operate the equipment, the B.B.C. for their co-operation, and Mr. J. A. Ratcliffe and Mr. I. J. Shaw, of the Cavendish Laboratory, Cambridge, for helpful discussions on the experimental programme.

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Heavy Beam Loading in Linear Electron Accelerators

By K. JOHNSEN †
Electrical Engineering Department, Imperial College, London

Communicated by D. Gabor; MS. received 12th April 1951

ABSTRACT. The influence of the beam current on the length, the obtainable particle energy and the efficiency of a linear accelerator is considered. The beam loading is especially important if the accelerator waveguide has low losses. The efficiency, defined as the ratio of H.F. power transformed into kinetic energy to the total H.F. power fed to the accelerator, can have a high value, 70–80%, but the particle energy obtainable drops considerably with heavy beam loading.

§ 1. INTRODUCTION

In theoretical investigations of linear accelerators it is common to neglect the effect of the beam loading on the accelerator performance. This is justified when only the field configuration is being considered. However, the influence of the beam loading on the attenuation, and consequently on the obtainable particle energy and on the accelerator length may well be appreciable. This is especially important in the case of low-loss accelerator tubes and it may be necessary, for instance, in dielectric loaded tubes (Shersby–Harvie 1948 b) to take the beam loading into account even for fairly small beam currents. Further, high beam currents are essential for some applications, e.g. in accelerators for medical purposes from which a high x-ray yield is required, and in accelerators for electron sterilizing experiments on food, etc.

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* Huxley and Ratcliffe state that " Under some important conditions the assumptions which allow us to replace equation (2) by equation (3) (i.e. $\nu/(\nu^2+p_D^2)$ by ν/p_D^2) are not valid ".

† Now at Chr. Michelsens Institute, Bergen.

In this paper the influence of beam loading will be investigated. In the calculations it is assumed that the field configuration is unchanged by the loading, an assumption which is valid, as already mentioned, in all practical cases, and which has been confirmed by a separate investigation.

As the electron velocity is close to the velocity of light along most of the accelerator, we shall confine ourselves to the case in which waveguide parameters are unchanged along the accelerator. The calculations are, therefore, not strictly correct for the bunching section, but this will hardly affect the overall performance of the accelerator.

§ 2. ESSENTIAL ACCELERATOR PARAMETERS

In this section we shall briefly summarize the main parameters which determine the operation of a linear accelerator.

The waveguide used as accelerator tube is best characterized by its equivalent shunt impedance per unit length and its equivalent series impedance. The shunt impedance per unit length is defined as

$$\eta = E^2/w \qquad \ldots\ldots(1)$$

where E is the amplitude of the axial electric field on the axis of the waveguide and w is the H.F. power dissipation per unit length, when there is no beam loading. (Sometimes different definitions are used, but there is always a simple relation between them.) Approximate values for η have been worked out or found experimentally for the most commonly used waveguides (Shersby–Harvie 1948 a, b, Mullett and Loach 1948, Slater 1948), and therefore η will be assumed to be known.

The series impedance is defined as

$$\xi = E^2/P \qquad \ldots\ldots(2)$$

where E has the same meaning as earlier and P is the H.F. power flow. ξ is also assumed to be known, as it can be worked out fairly accurately for the waveguides in question. Walkinshaw (1948) has calculated it, for instance, in the case of the metal corrugated waveguide.

If an external feedback is used, it is characterized by the feedback ratio, which is defined as (Shersby-Harvie and Mullett 1949)

$$r = (P_0/W_0) - 1 \qquad \ldots\ldots(3)$$

where P_0 is the power flow at the input of the accelerator and W_0 is the power delivered by the H.F. source.

In the calculations the ratio E_1/E_0, where the indices 0 and 1 indicate the value at the input and output respectively, will be used as a parameter. If the feedback ratio and the losses in the feedback circuit are known this ratio can be worked out. If no feedback is used, it is determined by the amount of H.F. power one can afford to lose in a matching load, or by practical considerations which limit the accelerator length.

The degree of beam loading is most conveniently expressed by the dimensionless quantity

$$m=\eta I \cos \phi_0/E_0 \qquad \ldots\ldots(4)$$

where η and E_0 have the same meaning as earlier, I is the beam current and ϕ_0 is the angle by which the stable point precedes the peak of the wave. The physical significance of the quantity m becomes more clear if (4) is rewritten as $m=E_0 I \cos \phi_0/(E_0^2/\eta)$ from which it is seen that m is the ratio of power absorbed by the beam per unit length to waveguide losses per unit length, all at the input end of the accelerator.

§ 3. THE ACCELERATOR LENGTH

From the definition of the shunt impedance (1) the loss per unit length is $w=E^2/\eta$. In addition, the beam absorbs an amount of H.F. power

$$w_e=EI \cos \phi_0 \qquad \ldots\ldots(5)$$

per unit length. The total conversion of H.F. power per unit length is then $w_t=w+w_e=(1+\eta I \cos \phi_0/E)E^2/\eta$ or, taking (4) into account,

$$w_t=(1+mE_0/E)E^2/\eta. \qquad \ldots\ldots(6)$$

The series impedance is defined by (2) and this, inserted in (6), leads at once to the following differential equation for the power flow:

$$dP=-(\xi/\eta)[P+m(P_0P)^{1/2}]dz \qquad \ldots\ldots(7)$$

the solution of which is

$$z=(2\eta/\xi)\ln[(m+1)/(m+E/E_0)] \qquad \ldots\ldots(8)$$

where z is measured from the input end, where the power flow is P_0. The length of the accelerator is given by

$$\xi L/2\eta=\ln[(m+1)/(m+E_1/E_0)]. \qquad \ldots\ldots(9)$$

It is difficult to build a feedback bridge for feedback ratios much different from unity and the two most important cases are, therefore, this case and that in which no feedback is used at all. Two examples are shown in Figure 1. In curve A, no feedback is used, and it is assumed that 10% of the H.F. power is dissipated in the matching load, which gives $(E_1/E_0)^2=0{\cdot}1$. In curve B, the feedback ratio is assumed equal to unity and the losses in the feedback circuit are neglected. This gives $(E_1/E_0)^2=0{\cdot}5$.

Firstly we notice the fact pointed out by Shersby-Harvie and Mullett (1949), that a feedback reduces the accelerator length considerably, and secondly, the point which is of main interest in this connection, that the accelerator length decreases rapidly with increasing beam loading.

Often, especially if there is no feedback and the beam loading is small, the frequency sensitivity and other practical considerations may limit the accelerator length (Shersby-Harvie 1948 a). It may then be convenient to rewrite (9) as

$$E_1/E_0=(m+1)\exp(-\xi L/2\eta)-m \qquad \ldots\ldots(9\,a)$$

and to substitute this value for E_1/E_0 in the expressions given in the next two sections.

§ 4. THE PARTICLE ENERGY

The particle energy at a point z is given by

$$U=U_0+\int_0^z E\cos\phi_0\,dz, \qquad \ldots\ldots(10)$$

where U_0 is the injection energy of the electrons. If (7) and (2) are taken into account, this gives

$$U - U_0 = (2\eta/\xi)E_0 \cos\phi_0\{1 - (E/E_0) - m \ln[(m+1)/(m+E/E_0)]\}. \quad \ldots\ldots(11)$$

The particle energy at the output is given by the same expression when $E = E_1$. A more convenient expression, however, is obtained by substituting

$$E_0 = [\xi W_0(1+r)]^{1/2} \quad \ldots\ldots(12)$$

which is derived from (2) and (3). For the gained particle energy this gives

$$\frac{U_1 - U_0}{(W_0/\xi)^{1/2} 2\eta \cos\phi_0} = (1+r)^{1/2}\{1 - (E_1/E_0) - m \ln[(m+1)/(m+E_1/E_0)]\}. \quad \ldots(13)$$

This expression has been plotted in Figure 2 for the same two cases as earlier.

It is seen that the obtainable particle energy depends strongly on the beam loading when the length is assumed to vary according to Figure 1. Already at $m = 1$ the energy is only about half the theoretical value for zero beam loading.

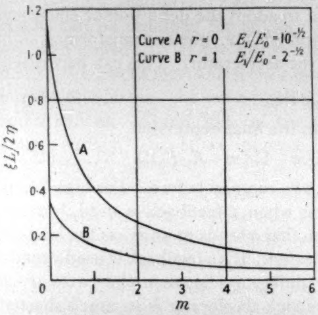

Figure 1. L = accelerator length, η = shunt impedance per unit length (ohm/m.), ξ = series impedance (ohm/m²), m = degree of beam loading $= \eta I \cos\phi_0/E_0$, where I is the beam current, E_0 the maximum accelerating field on the axis at the input, and ϕ_0 is the angle by which the stable point precedes the peak of the wave.

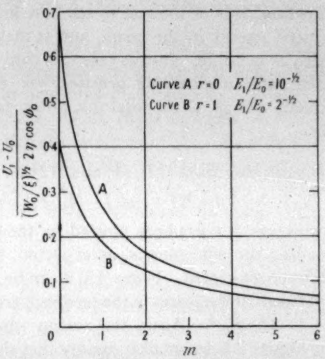

Figure 2. $U_1 - U_0$ = particle energy gain, W_0 = available H.F. power, m = degree of beam loading.

At first sight it may appear surprising that the curve in Figure 2 for the accelerator without feedback always lies above that for the accelerator with feedback ratio equal to one. The reason for this is mainly that the length, according to Figure 1, is much shorter for the feedback accelerator. However, from the definition of m it is clear that if we are interested in a given beam current, its value for the accelerator with feedback is $(1+r)^{-1/2}$ times that when no feedback is used. This means that, in this case, the accelerator without feedback has higher loading than that with feedback, and we shall not necessarily get higher particle energy by choosing an accelerator without feedback.

In addition to this comes the fact already pointed out at the end of the previous section, that an accelerator with no feedback and small beam loading often cannot be made as long as assumed in Figure 2. If, for this reason, L is fixed and not E_1/E_0, the dependence of the particle energy is, of course, smaller than found in the examples shown in Figure 2. In that case it is convenient to insert from (9 *a*) into (13). This gives, in the case of no feedback

$$\frac{U_1 - U_0}{(W_0/\xi)^{1/2} 2\eta \cos \phi_0} = (1+m)[1 - \exp(-\xi L/2\eta)] - m\xi L/2\eta, \qquad \ldots\ldots(13a)$$

which shows that $U_1 - U_0$ in that case is a linear function of m, whose slope depends on the parameter $\xi L/2\eta$ only. If this parameter is much smaller than unity the slope is small. From Figure 1 it is noticed that if the length has to be fixed smaller than the value given by curve A, $\xi L/2\eta$ is usually considerably smaller than unity.

§ 5. THE EFFICIENCY

The efficiency of a linear accelerator is defined differently according to the purpose for which the accelerator is designed. If as high particle energy as possible is required from the accelerator, a definition is chosen which is closely related to the shunt impedance per unit length (Shersby-Harvie 1948 a). Sometimes, however, it is of interest to see how much of the H.F. power is transformed into kinetic energy in the beam, and it may be of importance to make this figure as high as possible. In this case it is convenient to adopt the definition of efficiency current in engineering practice, viz. the ratio of H.F. power transformed into kinetic energy, to the total H.F. power fed to the accelerator. This can be written as

$$n = I(U_1 - U_0)/W_0. \qquad \ldots\ldots(14)$$

Substituting from (13), (4) and (12) we obtain the final expression

$$n = 2(1+r)m\{1 - (E_1/E_0) - m \ln [(m+1)/(m + E_1/E_0)]\}. \qquad \ldots\ldots(15)$$

In Figure 3, n has been plotted for the same two cases as before. For high beam loading the efficiency is, as expected, highest when a feedback is used, but the difference is small. From (15) it can be shown that n tends to unity, as the loading increases, if the losses in the feedback are neglected. If no feedback is used n tends to $1-(E_1/E_0)^2$. Again the reason why the difference between the two curves in Figure 3 is so small is mainly that the feedback accelerator is so much shorter than the accelerator without feedback. If the latter, for practical reasons, has to be made shorter than found from Figure 1, its efficiency drops accordingly, as seen from (9 *a*) and (15).

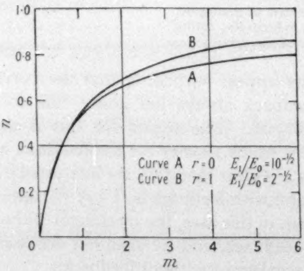

Figure 3. n = efficiency = $I(U_1 - U_0)/W_0$, m = degree of beam loading.

The interesting thing to notice is that the efficiency defined in this way increases so rapidly with m that there seems to be little difficulty in transforming most of the H.F. power into kinetic energy in the beam. If the accelerator is designed for the purpose, 70–80% should not be too optimistic a figure. However, as shown in the previous section, a high efficiency can be obtained only at the expense of the particle energy, if the power from the H.F. source cannot be correspondingly increased.

§ 6. CONCLUSIONS

The calculations in the previous sections have shown that in all respects the beam loading affects the performance of a linear accelerator, usually unfavourably, but beneficially for some applications, as shown in § 5. It is only if the degree of beam loading, defined by equation (4), is considerably smaller than unity that theories worked out for zero beam loading apply. To see more closely what this implies, a numerical example will be given.

Let the available H.F. power be 2 MW., the wavelength 10 cm. and $\phi_0 = 35°$. If a hole diameter of 4 cm. is used a reasonable value for η is about 30 MΩ/m. With this hole diameter it can also be shown that $\xi = 3$ MΩ/m². This gives $m \simeq 10\, I(1+r)^{-1/2}$, which shows that if there is no feedback, m reaches unity for a beam current of 100 ma. (in each pulse). If a unit feedback ratio is used, $m = 1$ when $I \simeq 140$ ma.

If we now assume that $r = 1$ and neglect the losses in the feedback, the following results can be obtained for an accelerator using the waveguide and H.F. source specified above: (i) If the beam current is neglected it is found from Figure 2 that $U_1 - U_0 \simeq 16$ MeV. and from Figure 1 that $L \simeq 7$ m. (ii) If the beam current is increased to 50 ma., the same figures have already decreased to $U_1 - U_0 \simeq 12$ MeV. and $L \simeq 5$ m. About 30% of the H.F. power is in this case transformed into kinetic energy. (iii) If we increase the beam current to about 300 ma., we get only $U_1 - U_0 \simeq 5$ MeV., $L \simeq 2$ m., but now n has increased to about 70%. The mean current in a linear accelerator can be up to about one thousandth of the peak current. From this example, it can therefore be concluded that beam currents usually required will not affect the performance of a linear accelerator very much, and that the theory worked out for zero beam loading can be used. That is true for all waveguides used in accelerators built so far. However, if waveguides can be made which have a much higher η, e.g. 5 to 10 times as high as for the all-metal waveguide, the beam loading must be taken into consideration even for much smaller currents than those assumed in (ii).

For some purposes, such as in the use of a linear accelerator as a neutron source or as an accelerator for sterilizing experiments on food etc., it may be essential to use the highest obtainable beam current. In these cases, the loading must be taken into account in the design of the accelerator, and the calculations carried out here indicate that satisfactory performances can be expected.

ACKNOWLEDGMENTS

The author wishes to express his thanks to the Electronized Chemicals Corporation, Norway A/S and the Royal Norwegian Council for Scientific and Industrial Research for making this investigation possible. The author is also indebted to Dr. D. Gabor for valuable advice, to Messrs. R. B. R.-Shersby-Harvie, W. Walkinshaw and L. B. Mullett for stimulating discussions, and to Mr. G. D. Sims for reading the manuscript.

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Currents in a High-Voltage Ion Accelerator Tube

By E. K. INALL
Research Laboratory, Associated Electrical Industries, Ltd.,
Aldermaston, Berks.

Communicated by D. R. Chick; MS. received 13*th April* 1951,
and in final form 27*th June* 1951

ABSTRACT. It is well known that a large electron current passes through a vacuum tube when used for the electrostatic acceleration of positive ions. This paper deals with experiments which show that many of the electrons are produced by neutral particles bombarding the lower end of the tube. The particles are formed in the region of the ion source canal and have a low energy due to the extraction potential. It has been found possible to suppress all the electrons produced below the bottom of the accelerator tube.

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§ 1. INTRODUCTION

When describing the performance of vacuum tubes for the electrostatic acceleration of ions, many authors have stated that the total current passed by the tube is from two to ten times greater than the ion current. Craggs (1942) showed that many fast neutral atoms enter the accelerator tube through the canal of a glow discharge ion source. However, the positive ion focusing efficiency of his accelerator tube (Craggs 1942) was such that a large part of the ion current was lost from the beam, therefore, although he concluded that most of the unidentified current was due to electrons produced by bombardment of the metal vacuum fittings at the base of the tube, he was unable to distinguish between secondary emission due to charged and uncharged particles.

The current in excess of the target current is a useless load on the generator supplying the accelerating voltage and the electrons give rise to a high intensity of x-rays. Also they are probably instrumental in causing breakdown in the tube.

Experiments described in this paper have been carried out on a tube with a high focusing efficiency and it was therefore possible to show that the excess current was mainly due to the bombardment of the base of the tube by neutral particles. This was confirmed by measuring the current passing through the tube when the ions were deflected back to the top of the tube immediately after entering it from the ion source canal. It was found also that nearly all the electrons originating below the last electrode could be prevented from entering the bottom of the accelerator tube.

§ 2. THE ACCELERATOR TUBE AND ION SOURCE

The accelerator tube consisted of three porcelain cylinders making a total length of 54 inches. In these there were twenty aluminium accelerating electrodes having an inside diameter of 7½ inches (Figure 1). Because of the short length of the tube, in comparison with the electrode apertures, practically all the particles from the ion source canal passed unobstructed through its full length.

The accelerating voltage applied to the tube was obtained from an electrostatic generator and could be set at any value up to 500 kv. A nearly uniform potential gradient along the tube axis was obtained by connecting the accelerating electrodes to suitable tapping points on a bleed resistor. A series of helical Perspex sleeves fitted outside the accelerator tube served to support the elements of this resistor, and also eliminated corona losses.

The ion source was similar to that described by Thonemann *et al.* (1948). The extractor voltage used was about 3·5 kv. except when it was varied to check the effect of this parameter on the electron current. The gas pressure in the discharge could be varied from 5×10^{-3} mm. Hg to 3×10^{-2} mm. Hg without producing much change in the pressure in the accelerator tube, while the pressure in the tube could be varied from 5×10^{-6} mm. Hg to 10^{-4} mm. Hg causing little change in the pressure within the ion source envelope. The proton content of the ion beam varied with time, but it was always between 80% and 90%.

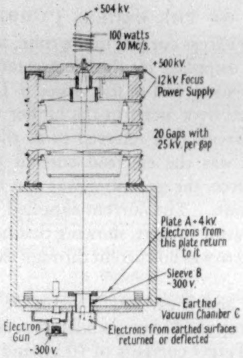
+504 kV.
100 watts 20 Mc/s.
+500 kV.
12 kV. Focus Power Supply
20 Gaps with 25 kV. per gap
Plate A +4 kV. Electrons from this plate return to it
Sleeve B -300 V.
Earthed Vacuum Chamber C
Electron Gun
-300 V.
Electrons from earthed surfaces returned or deflected

Figure 1. Accelerator focus electrodes and secondary electron suppressing electrodes.

The inside of the top plate of the tube was coated with willemite, so that electrons striking the plate produced a visible image which could be seen in a mirror placed in the vacuum chamber at the base of the tube.

§ 3. THE VACUUM CHAMBER AND SECONDARY ELECTRONS SUPPRESSING ELECTRODES

The porcelain cylinders were supported on a steel chamber (Figure 1) which was at ground potential. Mounted on insulators across the bottom of the chamber there was an aluminium plate A, with a 1-inch diameter hole in it to allow the beam to pass to the target below. A brass sleeve B was supported from the plate by means of an insulating flange. The inside diameter of B was 1½ inches so that particles from the anode of the tube could not strike it. When A and B were earthed high velocity particles striking A or earthed surfaces below B produced electrons, many of which were deflected into the accelerator tube by the weak field in the neighbourhood of A due to the potential on the lower accelerating electrode. If A were charged sufficiently positive, the field at its surface was such that electrons produced there were returned to it. However, electrons produced below A were accelerated through the central aperture when B was earthed. When B was charged to −300 volts with respect to earth no electrons passed up through it.

§ 4. MEASUREMENT OF TARGET CURRENT

The target was mounted 20 inches below plate A at the end of a 2-inch diameter tube made up of flexible bellows, brass tube and a length of glass tube.

The ion beam reaching the target was focused into a spot 3 mm. in diameter. In order to measure the ion current incident on the target it was necessary to ensure that the secondary electrons emitted returned to the target. This was done by supporting on the glass tube, ½ inch above the target, a large diameter metal ring which was charged to −200 volts with respect to the target. As a check on the effectiveness of this method of electron suppression the target current was observed with negative potentials of up to 1,000 volts applied to the ring. It was found that −100 volts returned approximately 85% and −200 volts returned approximately 95% of the electrons.

§ 5. MEASUREMENT OF THE EXCESS CURRENT IN THE TUBE

In order to measure the excess current in the tube, it was necessary to measure not only the total current supplied by the electrostatic generator, but also the currents down the bleed resistor and to the target. The difference between the generator current, measured by a meter in the higher voltage enclosure, and the resistor plus the target currents was referred to as the excess current, in other words the excess current was the electron current plus any unfocused ions. Before starting the ion source, the generator was run up to the voltage at which observations were to be made. The current supplied by the generator was then found to be equal to the resistor current, showing that no corona occurred from the accelerator tube and that there was no current through the tube when the ion source was off.

With the electrodes A and B earthed, a series of measurements was made using a constant beam energy of 350 kev., while the pressure in the tube was varied by throttling the pump. Target currents of 10 μa. and 20 μa. were used, and the ratio of excess current to target current was determined. The variation of this ratio with pressure is plotted in curve 1 of Figure 2. It was reduced to the values plotted in curve 2 by charging plate A to +4 kv. and sleeve B to −300 volts.

Thus, when plate A was earthed, the electrons leaving it produced in the tube a current represented by the difference between the corresponding ordinates of curves 1 and 2, while curve 2 represents the upper limit of the ion current lost from the beam, and striking plate A. Hence, it follows that the electron current was greater than twenty times the ion current lost.

With a fixed value of target current the excess current varied rapidly with the pressure in the envelope of the ion source. This is shown by the points *a* and *b* which lie off curve 1 of Figure 2, points *a* being for a lower pressure and *b* for a higher pressure than the value maintained for the other points on the curve. This will be discussed later.

When the electrodes A and B were earthed, the electrons produced a fairly uniform fluorescence over an area about 6 inches in diameter on the plate at the top of the tube. With the suppressing voltages applied there was no fluorescence when the pressure was less than 10^{-5} mm. Hg but there was faint glow at a pressure of 5×10^{-5} mm. Hg. Thus it was of value to know how much of the excess current represented by curve 2 was unfocused ions and how much was unsuppressed electrons. An x-radiation meter was therefore mounted beside the lower end of the tube in a position where x-rays from the top were detected after passing through

$\frac{1}{4}$ inch of steel. It was then assumed that all the excess current observed, when the tube pressure was 5×10^{-6} mm. Hg with the electrodes A and B earthed, was due to return electrons, and this was used to give a calibration of the radiation meter in terms of electron current. The radiation observed when the suppressing voltages were applied to A and B indicated that the electron current was negligible when the gas pressure was less than 10^{-5} mm. Hg, and one fifth of the target current when the pressure was 5×10^{-5} mm. Hg; that is, about one half of the excess current shown by curve 2 was due to electrons. The other half is assumed to be due to ions scattered from the beam during transit through the tube.

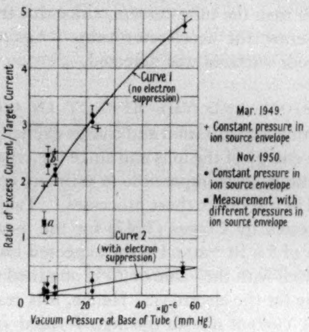

Figure 2.

Note.—Errors shown are due to limit of reading meters.

§6. POSSIBLE SOURCES OF THE ELECTRONS

As was stated above, the electron current was greater than twenty times the ion current lost from the focused beam. This large ratio indicates that all the electrons could not have been produced directly by ion bombardment (see Hill *et al.* 1939). However, they may have arisen in two ways. First, the ions incident on plate A released some electrons which may, as suggested by Craggs, have been multiplied, in their passage to the anode, by successive collisions with the electrodes. In fact this did not occur because the resistor current was unaltered by suppressing the electrons, even when the excess current and the resistor current were each twice the beam current. Also, the high x-ray flux produced after filtering due to $\frac{1}{4}$ inch of steel showed that the electrons must have been accelerated through more than half the total potential. Secondly, neutral particles entering the tube from the ion source may have liberated electrons from plate A. If neutral particles which entered the tube with an energy due to the extractor potential of the ion source were responsible for the production of the electrons, the current would have been independent of the beam energy. If, on the other hand, they were produced by the lost ions, followed by some unobserved multiplication, the magnitude of the effect would probably not have been independent of beam energy, although two compensating variations may have tended to make it so. The excess current was therefore measured with beam energies of 100 kev., 350 kev. and 450 kev. In each case the same current ratios were observed, a fact which is consistent with the theory that neutral particles gave rise to the electrons.

§7. ELECTRONS DUE TO NEUTRAL PARTICLES

To find how many electrons were produced by neutral particles, the ions were deflected back to the top of the tube immediately after entering it from the ion source. This was achieved by charging the top two electrodes to +4 kv. with respect to the extractor anode. Then when plate A was earthed through a meter, the current in the tube was equal to the electron current measured previously with the beam passing down the tube. These measurements were made over the limited pressure range of 5×10^{-6} to 2×10^{-5} mm. Hg. As would be expected, with plate A charged to suppress the electrons, there was no current through the tube. Thus only neutral particles could be responsible for the electrons. The current to plate A was 4% greater than the tube current, since this fraction of the secondary electrons did not re-enter the accelerator tube. Again, no multiplication of electrons at the electrode surfaces was detected.

§8. DISCUSSION, AND THE EFFECT IN OTHER TUBES

Since the cross section for ionization and charge exchange between the ions and the gas varies with the energy of the ions and since the cross section is known only over a limited energy range, it is impossible to estimate accurately the fraction of the beam which would be lost due to these processes. Taking the value of approximately 2 metres suggested by Craggs (1942) for the mean free path for charge exchange at a pressure of 5×10^{-5} mm. Hg the expected loss would be of the order of 30% which agrees well with the value of 20% obtained from curve 2, Figure 2, after allowance is made for the electrons. Hence, it is reasonable to assume that curve 2 represents the current due to the ions formed in the tube, by charge exchange and ionization, plus some secondary electrons which were not suppressed.

The shape of curve 1 can be explained approximately as follows. At a pressure of 5×10^{-6} mm. Hg neutral particles produce an electron current equal to twice the target current. At a pressure of 5×10^{-5} mm. Hg, 0·2 of the beam is neutralized by charge exchange so that in order to maintain a constant target current the beam from the ion source was increased to 1·2 times its value at 5×10^{-6} mm. Hg. Also the neutralized ions strike the bottom of the tube, giving a further increase, which cannot be estimated, in the electron current due to neutral particles. Thus, at a pressure of 5×10^{-5} mm. Hg, the electron current due to neutral particles would be slightly more than 2·4 times the target current. The ions formed in the tube would be mainly diatomic and consequently would produce about six electrons per ion (Craggs 1942) giving a further electron current of 1·2 times the current to the target. Therefore the total electron current expected at a pressure of 5×10^{-5} mm. Hg is slightly more than 3·6 times the target current. The value observed from curve 1 is 4·4 after allowing for the scattered ion current.

It is concluded that when the tube pressure is less than 10^{-5} mm. Hg, the excess current in a tube having good positive ion focusing is due to electrons produced at the base of the tube by neutral particles formed in the region of the ion source canal.

The neutral particles may be formed by protons being neutralized at the walls of the canal as Oliphant (1929) demonstrated in the case of helium. They are, however, more likely due to charge exchange with the gas in the canal, where the pressure is about 10^{-2} mm. Hg, and the mean free path is therefore about the same as the length of the canal. This would explain why the number increases with the pressure in the ion source, so displacing points *a* and *b* from curve 2, and why the beam current shows a corresponding drop.

If an arc type ion source is used, the neutral particles will have a very low energy and may not give rise to electrons.

A very great improvement in the performance of the accelerator can be realized by the suppression of the electrons. For example, the electrostatic generator used can only supply 160 μa. of current at 500 kv., of which 50μa. pass down the resistor. Originally, only 25 μa. of beam could be accelerated, but when the electrons were suppressed it was possible to increase this to 100 μa. At the same time the x-ray intensity in the target room was reduced from 1·0 r/8 hr. to less than 0·07 r/8 hr., and erratic movement of the beam was eliminated.

ACKNOWLEDGMENTS

The author wishes to thank Mr. Hancock and Mr. Cheetham for their assistance in this work, his colleagues for many helpful discussions, and Dr. T. E. Allibone for his interest and for permission to publish this paper.

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The Regenerative Deflector for Synchro-cyclotrons

By K. J. Le COUTEUR
Department of Theoretical Physics, University of Liverpool

Communicated by H. Fröhlich; MS. received 30th April 1951

ABSTRACT. Tuck and Teng have proposed to extract the beam from a synchro-cyclotron by modifying the normal magnetic field in such a way as to build up the amplitude of radial oscillations until protons can escape from the magnet. In the present paper the method is studied analytically and formulae are derived which express the performance in terms of the magnetic field perturbations: these formulae have been evaluated numerically in sufficient detail to guide practical design.

§1. INTRODUCTION

The new method recently proposed by Tuck and Teng (1950, 1951) for extracting the proton beam from a synchro-cyclotron promises an unusually high efficiency of operation. Their proposal is to modify the normal magnetic field at the edge of the cyclotron in such a way as to build up the amplitude of the usual radial oscillation of the protons about their mean orbit to a value sufficient to allow protons to escape from the cyclotron on the outward swings.

The arrangement is shown in Figure 1 (*a*). Normal synchro-cyclotron acceleration continues up to the radius r_s at which beam extraction is to commence. Outside r_s the vertical magnetic field H decreases with radius in a region, of angular width θ_p, called the 'peeler' and increases with radius in a region of angular width θ_q called the 'regenerator': these deviations of H from the value H_s of the normal field at r_s are shown in Figure 1 (*b*). Tuck and Teng have shown that the required fields can be produced by blocks of iron fixed between the poles of the electromagnet.

Tuck and Teng have proved the feasibility of their deflector by tracing the paths of particles through the system, in steps of a few degrees only, for several combinations of peeler and regenerator field strengths. The method is in principle precise, though in practice the lengthy graphical computation may introduce some numerical error. The disadvantage is that the procedure is too laborious to permit the examination of sufficient arrangements to determine the optimum values of the six disposable quantities, namely r_s, θ_p, θ_q, the angle d, and the peeler and regenerator field strengths, which enter into the design.

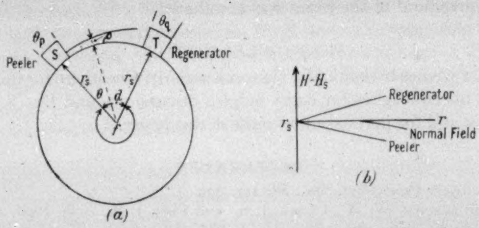

Figure 1. (*a*) Peeler–regenerator coordinates. (*b*) Radial variation of vertical field H from radius r_s outwards.

In connection with the design of a deflector for the Liverpool 156-in. synchrocyclotron, the problem has been studied by purely analytical methods which readily yield a complete solution if the particle equations of motion are linearized. This introduces only negligible errors and the analytical solutions in good agreement with the numerical values computed by Tuck and Teng in particular cases. The present method allows a general survey of the problem and is particularly well adapted to determine the best combination of angles and field strengths.

The main results and final formulae are in §§4, 5, 6 and 8; additional mathematical details are given in three Appendices.

§ 2. NEED FOR LINEAR RADIAL VARIATION OF DEFLECTOR FIELDS

The azimuth θ and vertical displacement z of a particle are measured from the centre of the peeler and from the median plane of the cyclotron respectively. If the magnetic field is represented by a vector potential A in the tangential direction, the Hamiltonian for the particle is

$$\mathscr{H} = (p_r^2 + p_z^2)/2m + (p_\theta/r - eA/c)^2/2m$$

where the angular momentum $p_\theta = mr^2 d\theta/dt + eA/c$ is a constant of the motion. Thus $(p_\theta/r - eA/c)^2$ is an effective potential for the motion in r and z and must be a minimum on the synchronous circular orbit of radius $r = cp/eH$.

Let ρ and z denote small radial and vertical displacements of the particle from its synchronous orbit. It is desired to build up a radial oscillation ρ of large amplitude without at the same time increasing the vertical oscillation, which would tend to reduce the intensity of the extracted beam. This can be achieved if the development of the effective potential in powers of ρ and z contains no terms in ρz, $\rho^2 z$ or ρz^2 which couple the radial and vertical oscillations. Mathematically this requires that the derivatives $\partial^3 A/\partial r^2 \partial z$ and $\partial^3 A/\partial r \partial z^2$ vanish on the synchronous orbit; these conditions are better stated as $\partial^2 H_r/\partial r^2 = 0$ and $\partial^2 H_r/\partial r \partial z = 0$, where $H_r = \partial A/\partial z$ is the radial component of the magnetic field. The first condition is automatically satisfied since by symmetry H_r vanishes everywhere on the median plane, the second is more conveniently written in terms of the vertical field component H as $\partial^2 H/\partial r^2 = 0$. Thus (Tuck and Teng 1950) *to minimize the coupling between the induced radial and the vertical oscillations the strong peeler and regenerator fields must vary linearly with radius as indicated in Figure* 1 (*b*).

When this condition is satisfied it is sufficient to use the first order Kerst–Serber (1941) equations of motion for ρ and z; these are

$$\frac{d^2\rho}{d\theta^2} + \left(1 + \frac{r}{H}\frac{\partial H}{\partial r}\right)\rho = 0, \qquad \frac{d^2 z}{d\theta^2} - \left(\frac{r}{H}\frac{\partial H}{\partial r}\right) z = 0, \qquad \ldots\ldots(1)$$

and may be derived from the above Hamiltonian or from the Lorentz equations of motion.

§ 3. SOLUTION OF THE EQUATIONS OF MOTION

The equations (1) must now be applied to the motion of a particle through the deflector, starting from the revolution at which it first enters the deflector fields with synchronous radius cp/eH equal to r_s. Then as indicated in Figure 1 (*a*), ρ is to be understood as the radial displacement of a particle measured from r_s, the increase (of the order of 0·0025 inch per turn) of the synchronous radius during the few (of the order of 10) cycles needed for extraction of the particle being neglected in comparison with the displacements ρ, of several inches, produced by the deflector.

If the unperturbed magnetic field in the neighbourhood of r_s varies as r^{-n} and its second radial derivative is neglected, equations (1) take the more familiar form

$$\frac{d^2\rho}{d\theta^2} + \Omega^2\rho = 0, \qquad \frac{d^2 z}{d\theta^2} + \omega^2 z = 0, \qquad \ldots\ldots(2\,a)$$

where

$$\Omega^2 = 1 - n \text{ and } \omega^2 = n \qquad \ldots\ldots(2\,b)$$

are the characteristic frequencies, measured in 'cycles per orbital revolution', of the radial and vertical oscillations in gaps d and f. Similarly, for the motion through the peeler

$$d^2\rho/d\theta^2 = (p^2 - 1)\rho, \quad d^2 z/d\theta^2 + p^2 z = 0. \qquad \ldots\ldots(3\,a)$$

with

$$p^2 = \{-(r/H)\partial H/\partial r\}_{\text{peeler}} \qquad \ldots\ldots(3\,b)$$

representing the logarithmic derivative of the total field in the peeler region and for the motion through the regenerator

$$d^2\rho/d\theta^2 + (q^2 + 1)\rho = 0, \quad d^2 z/d\theta^2 = q^2 z \qquad \ldots\ldots(4\,a)$$

with

$$q^2 = \{(r/H)\partial H/\partial r\}_{\text{regen}} \qquad \ldots\ldots(4\,b)$$

representing the logarithmic derivative of the total field in the regenerator region.

Because of the assumed form (Figure 1 (*b*)) of the deflector fields, these equations (3 *b*), (4) apply only if the displacements in peeler and regenerator are outward. In practice the particles approach the deflector with the radial oscillation, of amplitude perhaps half an inch, induced during the previous acceleration and the outward maximum of this oscillation precesses in azimuth at the frequency $1 - \Omega \simeq 0{\cdot}03$ cycles/rev. Deflection of a particle begins when this precession brings the outward swing of its existing radial oscillation into the region occupied by the peeler and regenerator fields. The expansion of synchronous radius during a complete cycle of precession is only about $30 \times 0{\cdot}0025 = 0{\cdot}075$ in., so that every particle may be assumed to pass through the conditions stated at the beginning of this section.

The solutions of these equations are most conveniently written in terms of the 2×2 transfer matrices **P**, **D**, **Q**, **F** which give the radial displacement and velocity at exit from peeler, gap d, regenerator and gap f respectively in terms of their values at entry to these structures and of transfer matrices P, D, Q, F which describe the vertical motion in the same way. Typical examples are, for gap d,

$$\begin{pmatrix}\rho\\ \dot\rho\end{pmatrix}_{\text{exit}} \equiv \mathbf{D}\begin{pmatrix}\rho\\ \dot\rho\end{pmatrix}_{\text{entry}} = \begin{bmatrix}\cos\Omega d^* & \dfrac{1}{\Omega}\sin\Omega d^*\\ -\Omega\sin\Omega d^* & \cos\Omega d^*\end{bmatrix}\begin{pmatrix}\rho\\ \dot\rho\end{pmatrix}_{\text{entry}} \qquad \ldots\ldots(5)$$

where $d^* = d - \tfrac{1}{2}(\theta_p + \theta_q)$; and for the regenerator

$$\begin{pmatrix}z\\ \dot z\end{pmatrix}_{\text{exit}} \equiv Q\begin{pmatrix}z\\ \dot z\end{pmatrix}_{\text{entry}} = \begin{bmatrix}\cosh q\theta_q & \dfrac{1}{q}\sinh q\theta_q\\ q\sinh q\theta_q & \cosh q\theta_q\end{bmatrix}\begin{pmatrix}z\\ \dot z\end{pmatrix}_{\text{entry}}. \qquad \ldots\ldots(6)$$

Then the resultant effect of one revolution through the deflector is expressed by the relationships ($\dot\rho = d\rho/d\theta$, $\dot z = dz/d\theta$)

$$\begin{pmatrix}\rho\\ \dot\rho\end{pmatrix}_{m+1} = \mathbf{A}\begin{pmatrix}\rho\\ \dot\rho\end{pmatrix}_{m}, \quad \mathbf{A} = \mathbf{FQDP} \qquad \ldots\ldots(7\,a)$$

and

$$\begin{pmatrix}z\\ \dot z\end{pmatrix}_{m+1} = A\begin{pmatrix}z\\ \dot z\end{pmatrix}_{m}, \quad A = FQDP \qquad \ldots\ldots(7\,b)$$

between the coordinates of the particle at the $(m+1)$th and mth entry to the peeler. The effect of m revolutions,

$$\begin{pmatrix}\rho\\ \dot\rho\end{pmatrix}_{m} = \mathbf{A}^m\begin{pmatrix}\rho\\ \dot\rho\end{pmatrix}_{0}, \quad \begin{pmatrix}z\\ \dot z\end{pmatrix}_{m} = A^m\begin{pmatrix}z\\ \dot z\end{pmatrix}_{0}, \qquad \ldots\ldots(8)$$

can easily be worked out by reducing **A** and A to their principal axes.

Each of the above matrices **D**, Q etc. has determinant unity and therefore the same is true of the resultant matrices **A** and A, defined by (7). If follows that the product of the eigenvalues of **A** is unity and the product of the eigenvalues of A is unity.

Let the eigenvalues and corresponding eigenvectors of A be e^{Λ}, (u, v) and $e^{-\Lambda}$, (u', v') so that

$$\mathbf{A}\begin{pmatrix}u\\ v\end{pmatrix} = e^{\Lambda}\begin{pmatrix}u\\ v\end{pmatrix} \quad\text{and}\quad \mathbf{A}\begin{pmatrix}u'\\ v'\end{pmatrix} = e^{-\Lambda}\begin{pmatrix}u'\\ v'\end{pmatrix}. \qquad \ldots\ldots(9)$$

Then since any initial radial displacement ρ_0 and velocity $\dot\rho_0$ can be expressed as

$$\begin{pmatrix}\rho\\ \dot\rho\end{pmatrix}_{0} = x\begin{pmatrix}u\\ v\end{pmatrix} + y\begin{pmatrix}u'\\ v'\end{pmatrix} \qquad \ldots\ldots\ldots(10)$$

with

$$x = \frac{\rho_0 v' - \dot\rho_0 u'}{uv' - vu'} \quad\text{and}\quad y = \frac{\rho_0 v - \dot\rho_0 u}{vu' - v'u},$$

the effect of m revolutions is derived from (8) as

$$\begin{pmatrix}\rho\\ \dot\rho\end{pmatrix}_{m} = \mathbf{A}^m x\begin{pmatrix}u\\ v\end{pmatrix} + \mathbf{A}^m y\begin{pmatrix}u'\\ v'\end{pmatrix} = xe^{m\Lambda}\begin{pmatrix}u\\ v\end{pmatrix} + ye^{-m\Lambda}\begin{pmatrix}u'\\ v'\end{pmatrix}. \qquad \ldots\ldots(11)$$

The deflector must be built so that one of the eigenvalues, say e^{Λ}, is greater than one and then the other $e^{-\Lambda}$ is necessarily less than one. After a few revolutions $e^{-m\Lambda}$ is negligible and

$$\rho_m = xe^{m\Lambda}u, \quad \dot\rho_m = xe^{m\Lambda}v\,. \qquad \ldots\ldots(12)$$

Thus, *independently of the initial conditions of entry into the deflector, if the eigenvalues of* $\mathbf{A}$ *are real the particle is captured into a motion which grows in radial amplitude by the factor* e^{Λ} *per turn, and has the phase of the characteristic oscillation* (u, v).

It is, of course, necessary to check that, in the characteristic oscillation (u, v), the radial displacement in the peeler and regenerator is outward.

A similar analysis may be applied to the vertical motion, with z, $\dot{z}$, A replacing ρ, $\dot{\rho}$, $\mathbf{A}$. Then if either of the eigenvalues of A is numerically greater than one the amplitude of the vertical motion will increase steadily; to avoid this both of the eigenvalues must have modulus unity since their product was shown to be one. *Thus the vertical motion remains finite after any number of turns if and only if the eigenvalues of A are of modulus unity, in which case they take the form* $e^{\pm i\lambda}$.

§ 4. CHARACTER OF THE INDUCED OSCILLATIONS

After capture in the deflector, the radial motion at the *m*th passage through the gap d has the form (Figure 1 (*a*))

$$\rho = ke^{m\Lambda}\sin(\psi + \Omega\theta) \qquad \ldots\ldots(13)$$

where ψ is a phase angle determined in Appendix B from the eigenvector (u, v) of $\mathbf{A}$ and k depends on the conditions of entry. Since ρ must be positive at both peeler $(\theta = 0)$ and regenerator $(\theta = d)$, the peeler regenerator angle must be less than π; in practice values less than $\pi/2$ are considered.

If the angular widths θ_p, θ_q of the deflector fields are small, (say $<20°$), the radial gain e^{Λ} and phase ψ are given (see Appendix B) by

$$\cosh\Lambda = \cos\Omega(f+d) - \frac{T-S}{2\Omega}\sin\Omega(f+d) - \frac{TS}{2\Omega^2}\sin\Omega f\sin\Omega d \qquad \ldots\ldots(14)$$

and

$$\cot\psi = \frac{e^{\Lambda} - \cos\Omega(f+d) - (T/\Omega)\sin\Omega f\cos\Omega d}{\sin\Omega(f+d) - (T/\Omega)\sin\Omega f\sin\Omega d} \qquad \ldots\ldots(15)$$

where $\Omega^2 = 1 - n$ and, for narrow fields, S and T are the integrated gradients of the field modifications* introduced at peeler and regenerator,

$$S = -\theta_p\{(r/H)\partial H/\partial r\}_{\text{peeler}}, \quad T = \theta_q\{(r/H)\partial H/\partial r\}_{\text{regen}}. \qquad \ldots\ldots(16)$$

The slight alterations required to deal with wide deflector fields are detailed in § 8 below.

Similarly stability of the vertical motion requires (see Appendix A, equation (13)) that

$$\cos\lambda = \cos\omega(d+f) + \frac{T-S}{2\omega}\sin\omega(d+f) - \frac{TS}{2\omega^2}\sin\omega f\sin\omega d, \qquad \ldots\ldots(17)$$

should be numerically less than unity.

Although (17) ensures that the vertical motion does not increase indefinitely, some vertical expansion of the beam is inevitable and this must be limited. Let $2h_0$ be the vertical thickness of the beam at entry into the deflector, then the maximum vertical displacement which occurs in the deflector for any particle is (see (A 10), (A 11))

$$h_{\max}{}^2 = h_0{}^2(1+\cos\epsilon)/(1-\cos\epsilon), \qquad \ldots\ldots(18)$$

and the root mean square thickness $2\bar{h}$ is given by (A 13) as

$$\bar{h}^2 = h_0{}^2/\sin\epsilon. \qquad \ldots\ldots(19)$$

---

* In a synchro-cyclotron, the normal field gradient is very small so that the derivatives (3*b*), (4*b*) of the total field are hardly distinguishable from the derivatives (16) of the field modifications. The distinction would be important in application to a machine, such as a synchrotron, with a large normal field gradient.

The vertical defocusing parameter ϵ is given by

$$\sin\epsilon = \frac{\sin\lambda\sin\omega(f+d)}{1-\cos\lambda\cos\omega(f+d)+(TS/2\omega^2)\sin^2\omega d}, \qquad \ldots\ldots(20)$$

which is derived in Appendix A.

Values of the parameters Λ, ψ, $\cos\epsilon$, which fully characterize the motion, have been calculated as functions of the peeler regenerator strengths S and T and of the angle of separation d and may be read from the graphs (Figure 4).

§ 5. REQUIRED RADIAL GAIN AND PHASE ANGLE

To escape completely from the magnetic field, the beam must pass through a magnetic channel in which H is reduced. The channel may be arranged as indicated in Figure 2, and Figure 3 shows a cross section of the mouth of the channel.

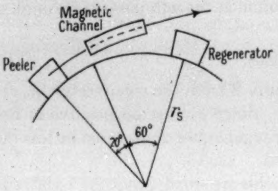

Figure 2. Possible magnetic channel arrangement.

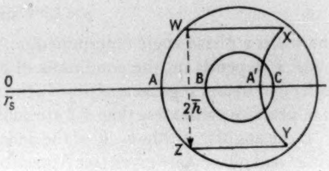

Figure 3. Cross section of channel entry, showing necessary radial gain $e^{\Lambda} = OA'/OA$.

In the last turn before entry into the channel, the radial displacement ρ must increase by more than the wall thickness, otherwise all particles hit the channel walls. For example, in Figure 3, a particle which at the *m*th revolution just grazes the outer wall at A must enter the channel at some point A′ on the next revolution. Then the radial gain,

$$e^{\Lambda} = \frac{OA'}{OA} = 1 + \frac{AA'}{OA} \geqslant 1 + \frac{AB}{OA}, \qquad \ldots\ldots(21)$$

must exceed the minimum value fixed by the wall thickness.

After sufficient revolutions, every particle in the beam must pass through the area AWXA′YZ in Figure 3, of height equal to the mean beam thickness $2\bar{h}$ given by (19), and of width WX = AA′ = ZY determined by the radial gain. The extraction efficiency is obviously given by the fraction of this area occupied by the channel entry BA′ and to achieve a good efficiency A′ must lie towards the extreme edge C of the channel entry.

These considerations indicate the desired radial gain e^{Λ}, values of about 1·4 should be sufficient in practice and the graphs show that these can be produced by realizable deflector fields.

The area WXYZ is equal to $2\bar{h}$.AA′ so the capture efficiency is inversely proportional to $\bar{h}$, that is, it is proportional to $\sqrt{\sin\epsilon}$.

An upper limit to the allowable value of the vertical defocusing parameter $\cos\epsilon$ is given by the requirement that $2h_{\max}$, given by (18), must be less than the dee separation. The initial thickness $2h_0$ is uncertain, but $\cos\epsilon = \frac{1}{2}$, giving $h_{\max} = 1{\cdot}7\,h_0$, should be tolerable. The graphs show that adequate radial deflections can be achieved without exceeding this defocusing.

The distance OA to the edge of the channel must be quite large, perhaps 3 in., otherwise, according to (21), an excessive radial gain would be required and there would also be a danger of the stray fields of the channel upsetting the normal orbits which should extend up to the radius r_s. If θ is the azimuth of the channel entry, (13) shows that particles will achieve a radial displacement

$$\mathrm{OA}/\sin(\psi+\Omega\theta) \qquad \ldots\ldots(22)$$

in the last turn before entering the channel. The permissible value of this displacement is limited by the room in the tank and more seriously by the fact that it tends to occur in the regenerator, so that the field gradient has to extend up to this distance. Thus $\sin(\psi+\Omega\theta)$ must be close to unity.

The azimuth θ of the channel entry fixes the direction of the extracted beam relative to the position of the peeler, and in typical buildings θ must be small if the beam is to emerge in an acceptable direction without the awkward necessity of placing the peeler structure inside the live dee.

These considerations indicate the desirable values of the phase ψ. With $\theta=20°$, giving reasonable clearance between peeler and channel as indicated in Figure 2, $\psi=50°$ giving $\sin(\psi+\Omega\theta)=\sin 70°=0{\cdot}93$ would be very satisfactory. The phase angle ψ is sensitive to the angle d between peeler and regenerator, the graphs show that suitable values can be realized.

§ 6. CONCLUSIONS

The graphs of Figure 4 cover all deflector strengths and angles which are likely to be of interest: additional points can be calculated from the formulae of §4.

Examination of the graphs drawn with $n=0{\cdot}0625$ and $d=30°$, $60°$, $90°$ shows that: (i) Given deflector strengths S, T produce greater deflections and more vertical defocusing at the larger peeler regenerator angles d. (ii) For any given angle d, the greatest ratio of radial gain to vertical defocusing occurs at the peaks of the $\cos\epsilon$ contours and this is the desirable combination of peeler and regenerator strengths. These peaks occur for peeler strengths weaker than the corresponding regenerator strengths because the peeler action is assisted by the normal field gradient. (iii) For field strengths chosen according to (ii) suitable phases ψ occur for $d=60°$, though a somewhat larger value would be satisfactory and may have to be used if it proves difficult to produce strong fields. For $d=90°$, the phase is rather too small. (iv) The ratio of radial gain to vertical defocusing decreases with d.

Comparison of the graphs drawn with $d=60°$ and $n=0{\cdot}0625$ and $n=0{\cdot}09$ shows that in the latter case slightly stronger fields are needed to produce a given gain e^{Λ}. Presumably this is because for $n=0{\cdot}0625$ the natural frequency $(1-n)^{1/2}$ of radial oscillation is closer to the frequency unity of the induced oscillation. Comparison of these two graphs also shows that the mathematical approximation made in §3, which amounts to neglecting the variation of n from point to point of the orbits, cannot introduce serious error.

The deflector strengths indicated by the graphs correspond to practicable fields. For example, for $d=60°$, $T=0{\cdot}5$, $S=0{\cdot}3$ would be satisfactory; with $r_s=75$ in., $H_s=18{,}000$ gauss these give $\theta_p(\partial H/\partial r)=-72$ gauss rad/in. and $\theta_q(\partial H/\partial r)=120$ gauss rad/in. which might be achieved by field gradients of -720 gauss/in. and 1,200 gauss/in., each acting for $0{\cdot}1$ radian.

§7. COMPARISON WITH RESULTS OF TUCK AND TENG

Tuck and Teng (1950) give results of computations for two deflectors with fields of angular width 0·1 radian:

	$(\partial H/\partial r)_{\text{peeler}}$	$(\partial H/\partial r)_{\text{regen}}$	S	T	d	Radial gain
II	− 1000 gauss/in.	750 gauss/in.	0·426	0·32	90°	1·3
III	− 1500 gauss/in.	900 gauss/in.	0·64	0·38	60°	1·5

the radial gain being estimated from their figure 8.15. For these values, and $n=0{\cdot}0625$, the graphs give $e^{\Lambda}=1{\cdot}3$ and $1{\cdot}4$ respectively, in reasonable agreement.

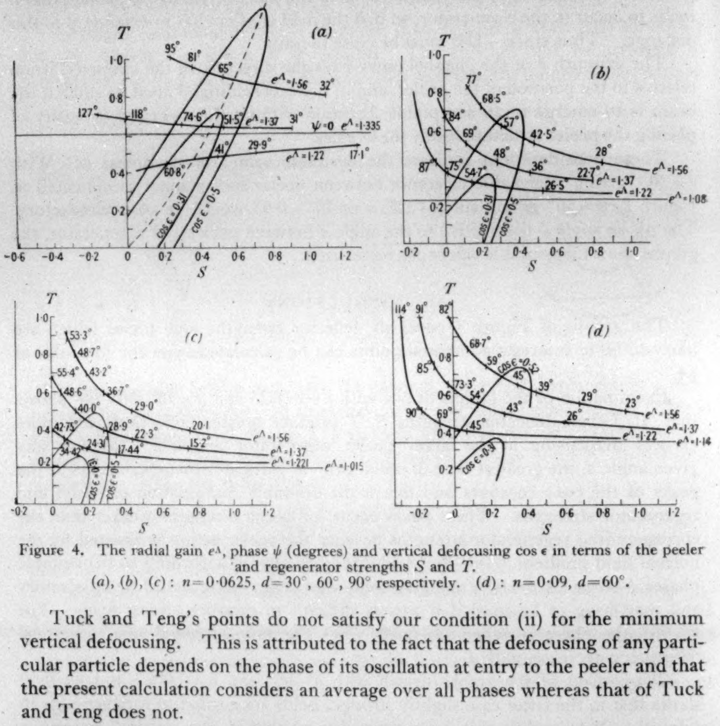

Figure 4. The radial gain e^{Λ}, phase ψ (degrees) and vertical defocusing $\cos\epsilon$ in terms of the peeler and regenerator strengths S and T.
(*a*), (*b*), (*c*): $n=0{\cdot}0625$, $d=30°$, $60°$, $90°$ respectively. (*d*): $n=0{\cdot}09$, $d=60°$.

Tuck and Teng's points do not satisfy our condition (ii) for the minimum vertical defocusing. This is attributed to the fact that the defocusing of any particular particle depends on the phase of its oscillation at entry to the peeler and that the present calculation considers an average over all phases whereas that of Tuck and Teng does not.

The present criterion (ii) indicates peeler strengths weaker than those required by Tuck and Teng. As the peeler field is difficult to make, this point may be important for construction of a working deflector.

§8. ALLOWANCE FOR WIDE PEELER–REGENERATOR FIELDS

Since only limited field gradients can be produced experimentally, it may turn out that the required deflector strengths S, T can most conveniently be achieved by using wide peeler and regenerator fields. In this case the simple definitions (16) must be replaced by (see Appendix C)

$$S=(p^2-n)\frac{\sinh\theta_p(p^2-1)^{1/2}}{(p^2-1)^{1/2}},\quad T=(q^2+n)\frac{\sin\theta_q(q^2+1)^{1/2}}{(q^2+1)^{1/2}},\quad \ldots\ldots(23)$$

for the radial motion and by

$$S=(p^2-n)\frac{\sin p\theta_p}{p},\quad T=(q^2+n)\frac{\sinh q\theta_q}{q} \qquad \ldots\ldots(24)$$

for the vertical motion.

Now $-p^2$ and q^2 were defined by (3*b*) and (4*b*) as logarithmic derivatives of the total field, $-n$ is the logarithmic derivative of the normal field. Therefore

$$(p^2-n)=-\{(r/H)\partial H/\partial r\}_{\text{peeler}} \quad \text{and} \quad (q^2+n)=\{(r/H)\partial H/\partial r\}_{\text{regen}} \qquad \ldots\ldots(25)$$

are precisely the logarithmic derivatives of the field modifications which previously appeared in (16). Equations (23) and (24) differ from (16) only by the factors $p^{-1}\sin p\theta_p$ etc., which modify S, T by $\pm\frac{1}{6}S^2\theta_p$ and $\pm\frac{1}{6}T^2\theta_q$ approximately; these differences are negligible for $S, T\simeq\frac{1}{2}$ and $\theta_p, \theta_q<\frac{1}{2}$.

With the definitions (23), equations (14) and (15) still describe the radial motion if f and d are replaced by $f'=f+(T\theta_q{}^2-S\theta_p{}^2)/24$, $d'=d+(T\theta_q{}^2-S\theta_p{}^2)/24$ to allow for the effective thickness of peeler and regenerator. Similarly, equations (17) and (20) still describe the vertical motion if f and d are reduced by $(T\theta_q{}^2-S\theta_p{}^2)/24$. These corrections to f and d are always negligible, so the graphs can be used for wide fields provided that S, T are interpreted according to (23) and (24).

§ 9. APPLICATION TO SYNCHROTRONS

The application of the regenerative deflector to synchrotrons has not been examined in detail. The analysis developed in this paper is applicable and suggests that in its present form the deflector will not work with the large value of n found in synchrotrons, but that a successful modification might be possible.

Consider $n=0{\cdot}6$ giving $\Omega=(1-n)^{1/2}=0{\cdot}63$ and $\omega=n^{1/2}=0{\cdot}78$ for the characteristic frequencies of radial and vertical oscillation: thus $2\pi\Omega=227°$ and $2\pi\omega=280°$. Then (14) gives

$$\cosh\Lambda=-0{\cdot}68+0{\cdot}57(T-S)-(TS/2\Omega^2)\sin\Omega f\sin\Omega d. \qquad \ldots\ldots(26)$$

Since $\cosh\Lambda$ greater than unity is required for radial deflection it is best to choose $S=0$, that is to rely on the large normal field gradient for the peeler effect. Then (26) requires T greater than 2·9 for radial deflection to occur. However (17) gives

$$\cos\lambda=0{\cdot}18-0{\cdot}63T-(TS/2\omega^2)\sin\omega f\sin\omega d \qquad \ldots\ldots(27)$$

so that the vertical motion would be unstable with such a strong regenerator. Similar conclusions hold for $n=0{\cdot}5$ and $n=0{\cdot}7$. The reason is that the free vertical oscillations of frequency ω more nearly resonate with the frequency induced by the deflector than do the free radial oscillations: this situation is just the reverse of that in the synchro-cyclotron where n is small.

It may be possible with large values of n to design a regenerative deflector to build up the vertical oscillations of the particles and to deliver them to a magnetic channel placed above the normal position of the circulating beam. At the same time the radial oscillations must not increase. This change in the role of the radial and vertical motions requires only obvious modifications of the analysis: the most important are that in (14) $\cosh\Lambda$ must be replaced by $\cos\Lambda$ which is required to be numerically less than unity and that in (17) $\cos\lambda$ must be replaced by $\cosh\lambda$, where e^λ is the 'vertical gain per turn'.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge my indebtedness to the detailed discussion of deflector design by Dr. J. L. Tuck and Dr. L. C. Teng. I would also like to express my thanks to Miss A. Rhys for most of the numerical calculations of this paper.

APPENDIX A

THE VERTICAL MOTION

For narrow deflector fields the peeler and regenerator matrices, introduced in § 3, take the simple form

$$P = \begin{pmatrix} 1 & 0 \\ -S & 0 \end{pmatrix}, \qquad Q = \begin{pmatrix} 1 & 0 \\ T & 0 \end{pmatrix}, \qquad \ldots\ldots\text{(A1)}$$

with S, T given by (16). Then

$$A = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \qquad \ldots\ldots\text{(A2)}$$

with $\alpha = \cos\omega(d+f) - \frac{S}{\omega}\sin\omega(d+f) + \frac{T}{\omega}\sin\omega f\cos\omega d - \frac{TS}{\omega^2}\sin\omega f\sin\omega d$

$$\beta = \frac{1}{\omega}\sin\omega(f+d) + \frac{T}{\omega^2}\sin\omega f\sin\omega d$$

$$\gamma = -\omega\sin\omega(f+d) - S\cos\omega(f+d) + T\cos\omega f\cos\omega d - \frac{TS}{\omega}\cos\omega f\sin\omega d$$

$$\delta = \cos\omega(f+d) + \frac{T}{\omega}\cos\omega f\sin\omega d.$$

The eigenvalues $e^{\pm i\lambda}$ of A are determined by

$$\alpha + \delta = e^{i\lambda} + e^{-i\lambda} = 2\cos\lambda \qquad \ldots\ldots\text{(A3)}$$

which gives equation (17). The two eigenvectors are given by

$$A\begin{pmatrix} a \\ e^{\pm ib} \end{pmatrix} = e^{\pm i\lambda}\begin{pmatrix} a \\ e^{\pm ib} \end{pmatrix}. \qquad \ldots\ldots\text{(A4)}$$

Then, if the initial vertical displacement and velocity of a particular particle are expressed as

$$\begin{pmatrix} z \\ \dot z \end{pmatrix}_0 = C\begin{pmatrix} a\cos\phi \\ \cos(\phi+b) \end{pmatrix} = \tfrac{1}{2}Ce^{i\varphi}\begin{pmatrix} a \\ e^{ib} \end{pmatrix} + \tfrac{1}{2}Ce^{-i\varphi}\begin{pmatrix} a \\ e^{-ib} \end{pmatrix}, \qquad \ldots\ldots\text{(A5)}$$

where the phase angle ϕ is fixed by

$$(\dot z/z)_0 = (\cos b - \sin b\tan\phi)/a, \qquad \ldots\ldots\text{(A6)}$$

the motion after n turns is

$$\begin{pmatrix} z \\ \dot z \end{pmatrix}_n = C\begin{pmatrix} a\cos(\phi+n\lambda) \\ \cos(\phi+b+n\lambda) \end{pmatrix}. \qquad \ldots\ldots\text{(A7)}$$

The amplitude of vertical oscillation after n turns through the deflector is defined by

$$h_n{}^2 = z_n{}^2 + (\dot z_n/\omega)^2 \qquad \ldots\ldots\text{(A8)}$$

and may be calculated from (A7) as

$$h_n{}^2 = \frac{C^2}{2\omega^2}(1+a^2\omega^2)\{1+\cos\epsilon\cos 2(\phi+\mu+n\lambda)\}$$

where $$\sin\epsilon = \frac{2a\,\omega\sin b}{1+a^2\omega^2} \quad\text{and}\quad \tan 2\mu = \frac{\sin 2b}{\omega^2a^2+\cos 2b}. \qquad \ldots\ldots\text{(A9)}$$

Thus h_n is related to the amplitude h_0 at entry by

$$\frac{h_n{}^2}{h_0{}^2} = \frac{1+\cos\epsilon\cos 2(\phi+n\lambda+\mu)}{1+\cos\epsilon\cos 2(\phi+\mu)}. \qquad \ldots\ldots\text{(A10)}$$

If a family of particles all enter the deflector with given amplitude h_0, the greatest vertical displacement of any member is $h_{\max}$ given by

$$h_{\max}^2/h_0^2 = (1+\cos\epsilon)/(1-\cos\epsilon) \qquad \ldots\ldots(\mathrm{A11})$$

which is equation (18) above.

For any given particle, the mean square amplitude of oscillation averaged over many turns is

$$h_0^2/\{1+\cos\epsilon\cos 2(\phi+\mu)\} \qquad \ldots\ldots(\mathrm{A12})$$

which for different particles varies between $h_0^2/(1+\cos\epsilon)$ and $h_0^2/(1-\cos\epsilon)$, depending upon the phase of vertical oscillation at entry to the deflector. For some particles the mean amplitude of oscillation is increased by the deflector, for others it is reduced; this compensation is a consequence of Liouville's theorem. The geometric mean of these two limits is

$$\overline{h}^2 = h_0^2/(1-\cos^2\epsilon)^{1/2} = h_0^2/\sin\epsilon, \qquad \ldots\ldots(\mathrm{A13})$$

which was used in (19) above to represent the mean square amplitude of the whole beam.

It remains to determine the characteristic angle ϵ in terms of the deflector strengths. Equation (A4) gives

$$\sin b = \frac{a\sin\lambda}{\beta} = \frac{-\sin\lambda}{\gamma a} \qquad \ldots\ldots(\mathrm{A14})$$

and then (A9) yields $\sin\epsilon = 2\sin\lambda/(\beta\omega - \gamma/\omega)$ which leads to (20) if the values (A2) of β and γ are substituted.

APPENDIX B

THE RADIAL MOTION

For narrow deflector fields, the peeler and regenerator matrices are

$$\mathbf{P} = \begin{pmatrix} 1 & 0 \\ S & 0 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1 & 0 \\ -T & 0 \end{pmatrix}, \qquad \ldots\ldots(\mathrm{B1})$$

with S, T given by (16). Then

$$\mathbf{A} = \begin{pmatrix} \boldsymbol{\alpha} & \boldsymbol{\beta} \\ \boldsymbol{\gamma} & \boldsymbol{\delta} \end{pmatrix} \qquad \ldots\ldots(\mathrm{B2})$$

is given by equations (A2) with ω replaced by Ω and S, T replaced by $-S$, $-T$. Equation (14) for the radial gain follows immediately and in (9) one may set $u = \boldsymbol{\beta}$, $v = e^{\Lambda} - \boldsymbol{\alpha}$.

From (12) it follows that, at the *m*th exit from the peeler, the radial displacement and velocity are

$$\begin{pmatrix} \rho \\ \dot{\rho} \end{pmatrix} = xe^{m\Lambda}\begin{pmatrix} 1 & 0 \\ S & 1 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = xe^{m\Lambda}\begin{pmatrix} u \\ Su+v \end{pmatrix} = ke^{m\Lambda}\begin{pmatrix} \sin\psi \\ \Omega\cos\psi \end{pmatrix} \qquad \ldots\ldots(\mathrm{B3})$$

say, with $\cot\psi = (Su+v)/\Omega u = (S\boldsymbol{\beta} + e^{\Lambda} - \boldsymbol{\alpha})/\Omega\boldsymbol{\beta}$ which is given explicitly by (15). Equation (B3) leads immediately to (13).

APPENDIX C

REDUCTION OF THE DEFLECTOR FIELDS

The matrices **P**, **Q**, P, Q introduced in § 3 to describe the deflection produced by fields of any width may be simplified by an analysis similar to the reduction of a thick lens to its principal planes.

Let

$$\Pi(\phi)=\begin{pmatrix}\cos\Omega\phi & \Omega^{-1}\sin\Omega\phi\\ -\Omega\sin\Omega\phi & \cos\Omega\phi\end{pmatrix} \qquad \text{......(C1)}$$

represent the transfer matrix for the radial motion through azimuth ϕ. Thus, in (5), $\mathbf{D}=\Pi(d^*)$.

Then any of the deflector matrices such as (6) may be reduced to the simple form of (A1) or (B1), by means of the general formula

$$\begin{pmatrix}a & b\\ c & a\end{pmatrix}=\Pi(\tfrac{1}{2}\phi)\begin{pmatrix}1 & 0\\ x & 0\end{pmatrix}\Pi(\tfrac{1}{2}\phi), \qquad \text{......(C2)}$$

with the effective strength x given by $x=c+b\Omega^2$ ......(C3)

and a thickness ϕ given by

$$\Omega^{-1}\sin\Omega\phi+x\Omega^{-2}\sin^2\tfrac{1}{2}\Omega\phi=b. \qquad \text{......(C4)}$$

Thus $$\mathbf{P}=\Pi(\tfrac{1}{2}\phi_p)\,\mathbf{P}'\Pi(\tfrac{1}{2}\phi_p),\quad \mathbf{Q}=\Pi(\tfrac{1}{2}\phi_q)\,\mathbf{Q}'\Pi(\tfrac{1}{2}\phi_q), \qquad \text{......(C5)}$$

with $$\mathbf{P}'=\begin{pmatrix}1 & 0\\ S & 0\end{pmatrix},\quad \mathbf{Q}'=\begin{pmatrix}1 & 0\\ -T & 0\end{pmatrix} \qquad \text{......(C6)}$$

where S and T are given by (23). Because of the group property

$$\Pi(d^*)\Pi(\tfrac{1}{2}\phi)=\Pi(d^*+\tfrac{1}{2}\phi),$$

there results $$\mathbf{A}'=\Pi(f')\mathbf{Q}'\Pi(d')\mathbf{P}' \qquad \text{......(C7)}$$

where $\mathbf{A}'=\Pi(\frac{1}{2}\phi_p)\,\mathbf{A}\Pi(-\frac{1}{2}\phi_p)$ relates the radial coordinates of the particle at its $(m+1)$th and mth passage through the centre of the peeler, and

$$f'=f+\tfrac{1}{2}(\phi_p+\phi_q-\theta_p-\theta_q),\quad d'=d+\tfrac{1}{2}(\phi_p+\phi_q-\theta_p-\theta_q). \qquad \text{......(C8)}$$

f' and d' are given explicitly by equation (25).

Actual calculations are much simplified by working with the matrices $\mathbf{P}'$, $\mathbf{Q}'$, $\mathbf{A}'$ and all the formulae have been derived in this way.

A similar reduction applies to the vertical motion, Ω being replaced by ω. In place of (C6) there results

$$P'=\begin{pmatrix}1 & 0\\ -S & 0\end{pmatrix},\quad Q'=\begin{pmatrix}1 & 0\\ T & 0\end{pmatrix} \qquad \text{......(C9)}$$

with S and T given by (24).

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Tuck, J. L., and Teng, L. C., 1950, *Institute of Nuclear Studies* 170, *Synchro-cyclotron Progress Report* III (University of Chicago), Chap. VIII; 1951, *Phys. Rev.*, **81**, 305.

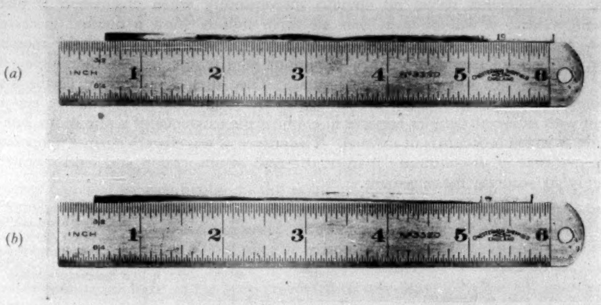
LETTERS TO THE EDITOR

The Lüders Deformation of Mild Steel

The writer has presented evidence (Hall 1951) which seemed to show that the Lüders bands appearing during the deformation of mild steel can each be characterized by a uniform shear front which propagates uniformly along the specimen. The shear displacement at the edge of the band is straightened by the grip constraints, and in addition to these two modes of deformation, shear and bending, a certain amount of creep extension also appears to be present.

In a tensile test at room temperature, the shear displacement will be straightened *plastically*, for the yield stress of the material which has just sheared over at the edge of the Lüders band is certainly no greater than the applied (lower yield) stress. Thus, a specimen removed from the testing machine before the end of the Lüders extension appears straight, apart from a small angular kink of the order of 2° at the band edge.

The Figure (*a*), shows a plastically deformed thin strip specimen of mild steel laid beside a straight edge. The dimensions of the strip are $135 \times 2 \times \frac{1}{2}$ mm., and the wide front face is shown in the photograph. No evidence of bending can be seen.



Strip specimens of mild steel deformed (*a*) at room temperature; (*b*) at 200° c.
Strain rate : 0·045/min.

Recently, however, these experiments on the deformation of mild steel have been extended to temperatures in the ' blue-brittle ' range, around 200° c. Here the deformed area of the specimen just behind the Lüders front ages very rapidly during the test, so that the yield stress in this region is much above the applied stress. Thus, the bend induced in the specimen is only straightened *elastically*, and on removing the specimen from the machine, a marked curvature remains in the test piece.

(*b*) shows a specimen with the same mechanical properties as (*a*), deformed at the same strain rate, but at 200° c. instead of at room temperature. A distinct curvature is seen in the plane of the wide front face of the specimen. Thus, mild steel may be bent by straining it in tension.

The radius of curvature of the bent specimen is very nearly 1 metre, so that there is a macroscopic strain inhomogeneity of about 0·2% between the two sides of the specimen. The inability of large specimens to accommodate such bending moments must undoubtedly play a part in determining the form of the Lüders band in the test pieces, and is perhaps the reason why in specimens of large cross section under a high load, the Lüders bands present a very complex appearance, while in these thin strips or wire specimens, usually only one or at most two Lüders fronts are observed (Hall 1951).

This work forms part of a project on the deformation of mild steel in the blue-brittle range, which was undertaken for the British Iron and Steel Research Association. The writer is indebted to the Association for permission to publish this letter. A full account of these experiments will be published elsewhere.

Cavendish Laboratory,
Cambridge.
10th October 1951.

E. O. Hall.

Hall, E. O., 1951, *Proc. Phys. Soc.* B, **64**, 742.

Ultrasonic Absorption in Supercooled Liquids

In December 1949 my attention was drawn to work on the existence of a discontinuity in the temperature coefficient of viscosity of di-phenyl ether in the critical region near the melting point (Dodd and Hu Pak Mi 1949). If this phenomenon be due to structural changes in the liquid, a marked discontinuity may be expected in a suitable function of the ultrasonic absorption in a liquid on its being supercooled.

Figure 1 shows preliminary results, obtained in this laboratory, for the ultrasonic absorption and wavelength in menthol at a frequency of 68 Mc/s. Figure 2 shows absorption only in salol and di-phenyl ether. I have termed 'classical' that part of the absorption which is due to the Stokes' shear viscosity term. This has been calculated from the measured wavelength and viscosity, the variation of density being ignored. While the measurements reveal no obvious discontinuity when absorption is plotted against temperature, it will be seen that all three liquids show a marked increase in absorption with approach to the freezing-point. It appears likely that the measured absorption in menthol passes through a minimum at a temperature rather higher than could be attained with the existing apparatus. The small temperature coefficient of absorption in di-phenyl ether makes the results for this liquid the least reliable: repeated experiment with different samples appears to confirm the existence of a minimum, but the temperature at which it occurs is ill-defined. The curves of wavelength against temperature give more evidence of discontinuity than in the case of absorption and lend themselves more readily to accurate measurement.

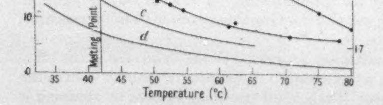


Figure 1.

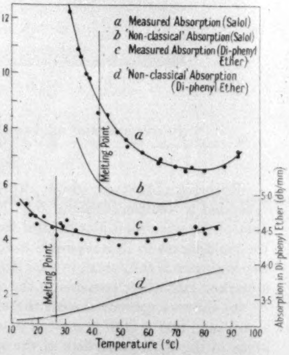


Figure 2.

It was thought that the observed increase in absorption on approach to the freezing point could be due to scattering at crystal nuclei. An experiment was performed in menthol at the higher frequency of 87·5 Mc/s. to test this hypothesis. At all temperatures the ratio of the measured absorption at the two frequencies used was close to 1·65, in accordance with a square law. If appreciable scattering were present an approach to 2·72 might be expected, in conformity with a fourth-power law.

The three liquids were chosen for convenience of melting point, ease of supercooling and availability of viscosity measurements made in them by Dr. Dodd and his co-workers. The frequency range used gives an ultrasonic attenuation of the order of 20 db/mm., so requiring only a few cm^3 of sample liquid. The pulse technique is used (Pellam and Galt 1946, Pinkerton 1949), with separate transmitter and receiver transducers and a six-inch fused quartz acoustic delay line (Rapuano 1947). Velocities are measured by injection of a phase-locked signal from a generator and observing the regular changes in output as the receiver is moved.

I am much indebted to Dr. Pinkerton for valuable advice with the design of the apparatus and to Dr. Dodd for viscosity data supplied prior to publication. It is hoped shortly to publish detailed results taken with a new apparatus nearing completion.

Physics Department,
University College of Leicester.
26th September 1951.

A. N. Hunter.

Dodd, C., and Hu Pak Mi, 1949, *Proc. Phys. Soc.* B, **62**, 454.
Litovitz, T. A., 1951, *J. Acoust. Soc. Amer.*, **23**, 75.
Pellam, J. R., and Galt, J. K., 1946, *J. Chem. Phys.*, **14**, 608.
Pinkerton, J. M., 1949, *Proc. Phys. Soc.* B, **62**, 286.
Rapuano, R. A., 1947, *Phys. Rev.*, **72**, 78.

The Optical Constants of Lead Sulphide and Lead Telluride in the Region 0·5–3 microns

In a recent communication (Paul, Jones and Jones 1951) measurements of the absorption characteristics of PbS in the wavelength range 2–12 microns have been described, and in addition the reflection coefficients of the material in this region have been measured. In this laboratory some experiments are in progress to measure the refractive and absorption indices of lead sulphide and related materials in the visible and infra-red spectral regions.

The method used depends on the measurement of certain reflection characteristics of specimens of the materials. The ratio of the reflection coefficients for incident light polarized in and perpendicular to the plane of incidence is measured for various angles of incidence, and, with the aid of graphs computed beforehand, the optical constants can be derived from these measurements. The method is an improvement on an earlier one due to Collins and Bock (1943), and it is hoped that complete details will be published shortly.

Measurements have so far been restricted to wavelengths below 2·6 microns, and have been made on polished surfaces of polycrystalline specimens of PbS, and on one polished cleavage face of an artificially prepared single crystal of PbTe.

The refractive indices and absorption constants for two specimens of PbS are shown in Figure 1. One of these specimens was pure Sardinian galena, and the other a piece of galena of unknown origin. The measurements in the visible region of the spectrum were made using a classical polarimetric method. The error in the refractive index values is some 4% or less, except at the extreme long wavelength end of the range, and is 10% in the absorption constant values.

Comparison of these absorption constant values with those of Gibson (1950) for chemically deposited layers of PbS, shows that for the present specimens the absorption edge occurs at a longer wavelength than for the layers, and that the long wavelength ‘ tail ’ to the edge, observed by Gibson, is very pronounced in this case. The magnitude of the absorption constant at wavelengths below 2 microns is considerably greater than for the chemically prepared layers.

Apart from a slight increase at about 0·8 micron, the refractive index shows no marked variation, rising gradually to values of about 4 at 2·5 microns, in agreement with the values quoted by Paul, Jones and Jones.

The surfaces of the specimens were first ground flat using successively finer grades of abrasive, and finally polished using a cloth lap and metal polish. The various specimens were ground and polished in as nearly as possible an identical manner, and the values obtained from four surfaces prepared separately agree to within 10–15%.

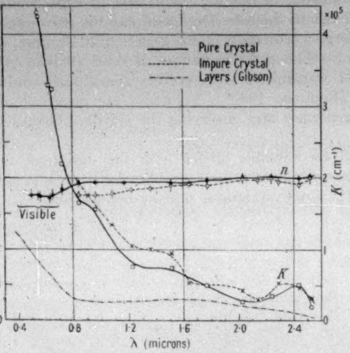


Figure 1. PbS, polished crystals.

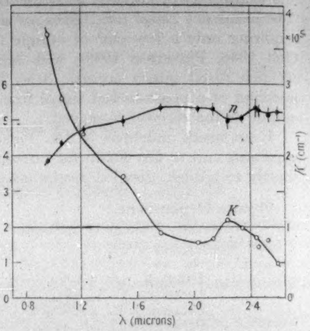


Figure 2. PbTe, polished single crystal.

The values of the refractive index and absorption constant for the polished single crystal of PbTe are shown in Figure 2. The refractive index rises from a value of 3·8 at 1 micron to 5·2 at 1·8 microns, remaining then roughly constant at about this value. The values of the absorption constant obtained are slightly higher than those obtained by Gibson for layers of the material prepared under various conditions.

Physics Department,
Royal Holloway College,
Egham, Surrey.
27th August 1951.

D. G. Avery*.

* Now at Telecommunications Research Establishment, Gt. Malvern, Worcs.

Collins, J. R., and Bock, R. O., 1943, *Rev. Sci. Instrum.*, **14**, 135.
Gibson, A. F., 1950, *Proc. Phys. Soc.* B, **63**, 756.
Paul, W., Jones, D. A., and Jones, R. V., 1951, *Proc. Phys. Soc.* B, **64**, 528.

The Refractive Index in Electron Optics

In 1933 Glaser derived a correct and now well-known value for the refractive index in electron optics. Although quite generally applicable, his is a particular value in so far as it omits an arbitrary function, and by a rational and indeed inevitable generalization suggested by Glaser's derivation, Opatowski (1943) obtained other values. Some of these, however, are wrong; and in 1949 we showed which of his generalizations are valid and how Glaser's approach must be modified in order to be theoretically sound and to lead always to correct results. As Glaser (1951) suggests that our discussion was based on misconceptions we should like to make a brief statement.

(*a*) As introduced into mechanics the Lagrangian L and the energy E are undefined to within, in general, different arbitrary constants. Opatowski saw, therefore, that the value $\mu=(L+E)/v$ as introduced by Glaser in 1933 has no better footing than the more general expression $\mu=(L+E+K)/v$ where K is an arbitrary constant. The relevant and in fact very simple point which Glaser does not seem to appreciate is that either expression is correct only if E and L are so normalized that the numerator equals $\Sigma\dot{q}_i\partial L/\partial\dot{q}_i$. This normalization is not automatically guaranteed and it is not explicitly stated by Glaser.

(*b*) That the time-dependent principle of Hamilton could not directly give a refractive index was stated clearly by de Broglie in his foundation papers on wave mechanics; he saw that Maupertuis' principle must be used. Glaser (1951) appears now to recognize this point but he makes no reference to the fact that in his original paper (Glaser 1933), which he still upholds, he refers to Hamilton's principle only. There he eliminates from L the time-dependent quantities by means of the energy integral so that $Ldt=\mu ds-Edt$; μ is a function of E, of the coordinates, via the electric and the magnetic potential, and of $\mathbf{s}$, the unit vector in the direction of the trajectory. Then he continues: "Hence (5) (i.e. $\delta\int Ldt=0$) becomes equivalent to $\delta\int \mu ds=0$ as (5) is not varied at the limits and thus $\delta\int Edt=0$." According to this statement $\int \mu ds$ must also be varied so that at the limits the time as well as the coordinates are kept constant. But the whole point of Fermat's and Maupertuis' principles is that the variation is carried out *regardless* of the time at the limits. Hence, the μ derived by Glaser is not a refractive index at all, and his claim of having derived it correctly is unfounded.

(*c*) Contrary to the statement of Glaser we made no suggestion either in connection with Opatowski's paper or otherwise that the general refractive index could not be specialized to a case of axial symmetry. Since this special ('isotropic') refractive index can be derived from the general refractive index, Glaser is not justified in calling the former a 'generalization' of the latter and his deduction from first principles can add as little to the elucidation of the situation as can the particular derivation of Maupertuis' principle he proposes.

Birkbeck College,
University of London.
14th September 1951.

W. Ehrenberg.
R. E. Siday.

Ehrenberg, W., and Siday, R. E., 1949, *Proc. Phys. Soc.* B, **62**, 8.
Glaser, W., 1933, *Z. Phys.*, **80**, 450; 1951, *Proc. Phys. Soc.* B, **64**, 114.
Opatowski, I., 1943, *Phys. Rev.*, **65**, 54.

The fact that the Lagrangian is definite with the exception of a total differential and the transition from Hamilton's principle to the principle of Maupertuis belong to the well-known fundamental theorems of analytical mechanics. We refer in this respect to any textbook. Therefore on this point a difference of opinion would be hardly possible. That our derivation (Glaser 1933) of refractive index in electron optics is based on these well-known principles, was fully shown in the paper quoted (Glaser 1951). However, we are not in the position to assume responsibility for the correctness of papers, published later by others, which claim to be based upon our work.

Technische Hochschule,
Vienna.
25th September 1951.

W. Glaser.

Glaser, W., 1933, *Z. Phys.*, **80**, 450; 1951, *Proc. Phys. Soc.* B, **64**, 114.

Obituary Notices

LESLIE JOHN COMRIE

THE death of Dr. L. J. Comrie, F.R.S., in London on 11th December 1950 at the age of fifty-seven has removed an original and independent figure from the ranks of British scientists.

Leslie John Comrie was born on 15th August 1893 at Pukekohe, Auckland, New Zealand. He was educated at Auckland Grammar School and Auckland University College, where, after a promising career, he graduated M.A. in 1916, with First Class Honours in Chemistry. He served in the First World War with the New Zealand Expeditionary Force, incurring severe wounds, with consequent amputation of a leg. It was doubtless this incapacitation, together with the handicap of increasing deafness, that caused him to reconsider his career, for after the war he remained in Britain as a New Zealand Expeditionary Force Scholar, studying astronomy and computation at St. John's College, Cambridge, where he was an Isaac Newton Student from 1920 to 1923. In 1923 a thesis, devoted to the history and detailed examination of methods of predicting the occultation of stars by planets, gained him the degree of Ph.D.

Two years in America followed, the first as Assistant Professor of Mathematics and Astronomy at Swarthmore College, Philadelphia, the second as Assistant Professor of Astronomy at Northwestern University, Evanston, Illinois. In 1925 he was appointed Deputy Superintendent of H.M. Nautical Almanac Office, Greenwich, and in 1926 entered upon his duties there. He was no stranger to Greenwich, having already, in 1920, spent two months at the Royal Observatory as part of the conditions of his Isaac Newton studentship. In 1930 he became Superintendent, a post which he held until 1936. Without delay he used the opportunity to embody in the next issue of the *Nautical Almanac*, for 1931, certain changes which he had without doubt considered for some years, notably the use of a standard equinox. The *Almanac* of 1931, breaking away from the tradition of almost a hundred years, incorporated, among other radical changes, the new ephemerides.

Comrie's imagination, essentially direct and pragmatic, had been captured, from his first contact with calculating machines, by their unexplored possibilities. He saw that with the rapid improvement in calculating machines many accepted procedures, legacies from the pre-machine era, were wasteful in time and effort and should be superseded or re-drafted in the newer idiom. He reconsidered such processes as interpolation, numerical integration and subtabulation from first principles, and devised simple and powerful repetitive techniques, requiring the minimum of copying and capable of being mastered and understood by operators of only moderate mathematical training. His ingenious mind ever busied itself with inventing methods for turning the latest devices, such as transfer levers, split keyboards or twin machines, to practical use. It was characteristic of him, for example, to harness together National Accounting Machines in order to build up and print tabular values from differences of high order, or to use punched cards and the Hollerith machine, as he did in 1928 with E. W. Brown's *Tables of the Moon*, to sum large numbers of harmonic terms. His methods, severely simple and simply described, are explicit in the prefaces or introductions to the various tables published under his editorship, notably in *Interpolation and Allied Tables*, reprinted from the *Nautical Almanac* for 1937.

In 1936 he resigned from the Superintendentship and began a new and independent venture, the founding on business lines of a computational firm, the now celebrated Scientific Computing Service Limited. This, at the time, was a bold step, though soon justified by increasing success, by requests for computational help on a large scale by the universities, by industry and by Government departments. With the outbreak of the Second World War the demand, official and unofficial, for such services was extraordinarily accelerated. The office in Bedford Square served as training ground for many computers, who entered with moderate qualifications and passed out as experts. By 1944 the staff numbered thirty; and after the war, with the business now firmly established and enlarged, Comrie found the need of an assistant manager, of similar interest and authority in all matters concerning computation. This was Dr. J. C. P. Miller, who now succeeds him.

Comrie had now for many years been recognized as an authority of international reputation on mathematical tables. He had been the first Director of the Computing Section of the British Astronomical Association, Secretary to the Mathematical Tables

Committee of the British Association, 1930-36, member of the Committee on Mathematical Tables of the National Research Council, Washington; and from 1948 to 1950 he was a member of the Mathematical Tables Committee of the Royal Society. He was elected a Fellow of the Royal Astronomical Society as early as 1919, and served on its Council from 1929 to 1932. He was also a Fellow of the Royal Statistical Society, and of the Physical Society. He was an active member of the International Astronomical Union, attending all its conferences from 1928 onwards, and was elected President, in 1932, of its Commission on Ephemerides. His native country recognized him by honorary fellowships, of the Astronomical Society of New Zealand in 1933 and of the Royal Society of New Zealand in 1945. In March 1950 he was elected a Fellow of the Royal Society, but lived only ten months to enjoy this culminating honour.

His papers, though they number about a hundred, represent only a small part of his work. A proportion of these, mostly appearing in the *Monthly Notices of the Royal Astronomical Society*, are on astronomical matters, but the larger proportion (often appearing in such journals as the *Office Machinery Users' Association Transactions*) deal with table-making and the application of various types of machine to problems of computation. Of great importance are his reviews and notes on errata in existing mathematical tables, a characteristic example being the article in *Mathematical Tables and Aids to Computation*, **1** (1945), pp. 391–399, on Jahnke and Emde's *Tables of Functions*. His erudition in this field was immense.

The tables for which he was personally responsible bear on every page the marks of his exacting standards, imposed not least upon himself. First, by rigorous systematic checking and counterchecking, the values must be impeccably accurate; they must also be interpolable with the maximum of convenience, readable with the maximum of ease, and typographically elegant. No one can doubt, perusing, for example, the third and fourth enlarged editions of *Barlow's Tables*, issued in 1930 and 1941, or, perhaps most conspicuously, the new *Six-Figure Mathematical Tables* in two volumes, logarithmic and natural, published by Chambers in 1948, that such aims have been achieved to the full. These last tables call for special mention; they resulted from the decision of the publishing firm, on the centenary of the old tables in 1944, to take the advice of a modern expert in regard to their re-issue. They are rightly described in the Editor's preface as " one of the greatest of all British table-making enterprises ".

In character Comrie was intolerant of imprecision, direct and outspoken to a degree sometimes startling, though this was really a concomitant of his essential simplicity and singlemindedness. The writer of the present notice was never a fellow committee-man with him; but it is no secret that his abruptness of manner in delivering an opinion, his refusal to use any of the ordinary arts of compromise, did not always meet with a favourable response. This was, however, only one side of the man, reflecting his inveterate honesty.

Perhaps one who was bound to Comrie by ties of friendship, of birth in New Zealand at about the same period, and of similar service and similar though less severe experience in the First World War, may conclude on a specially personal note. Comrie as a young man, before departing for France and suffering so severely, was physically vigorous and energetic, a good rifle shot and tennis player, pursuits in which his qualities of precision and decisiveness found a ready outlet. It is more than probable that the disabilities that permanently precluded such active outlets produced a certain frustration, which in its turn was overcome by the unremitting energy which Comrie poured into everything that concerned astronomy, machines, tables and computation; for, as has already been stated, the amount of work he did in his lifetime cannot by any means be summed up in terms of publications. Not least was the help he unobtrusively and willingly gave to others in the field, of which a fitting memorial exists in the *Index of Mathematical Tables* of Fletcher, Miller and Rosenhead, 1945, already classical. New Zealanders visiting London, and astronomers and computers from America and Europe can testify to the warm hospitality he gave them, ably seconded by his wife; but very few knew of his many acts of secret and private generosity, some of which have only come to light since his death. The picture that lingers most in the mind is of Comrie as host, cut off by deafness, it is true, from the cross-currents of conversation, but himself sitting quietly happy at the evident enjoyment of his guests. It is a question whether, in giving full rein to his proclivities, he did not drive himself too hard along a single track; but his work is of great, permanent and visible value, and he has set standards, in all that concerns table-making and computation, that can hardly be surpassed.

A. C. AITKEN.

LEONARD GEORGE GRIMMETT

I WORKED as assistant to L. G. Grimmett for 18 months, 1934–35, in the Physics Department of the Radium Beam Therapy Research at The Radium Institute, London. I have had as transient a contact with many others, yet they have passed out of my recollection and interest. In contrast, Grimmett made a lasting impression. I have always felt a surge of pleasure on meeting him again, or on hearing of his exploits, and was very grieved to learn of his sudden death from heart failure on 27th May 1951 in Houston, Texas. He had a unique and admirable personality.

The two pillars of Grimmett's career were his love of science and of music; he was a pianist of the first rank. On leaving the Holloway County Secondary School he paid the expenses of a physics degree course at King's College from his earnings in restaurant and theatre bands. He made a voyage as ship's pianist to Buenos Aires. Undoubtedly this double life was a heavy strain. To me he deplored that, at a time when he was engaged in this struggle, his Professor was completely discouraging, believing that a degree could not be taken in this eccentric fashion. Nevertheless it was taken (in 1926), and so well that he was encouraged to do three years' research. In 1930 he became Assistant Physicist at the Westminster Hospital Radium Annexe, working under Professor H. T. Flint, and together they developed one of the earliest hospital physics departments. Grimmett was a pioneer in the development of radium beam units for the treatment of malignant disease; a two-gramme unit designed by him in the early 1930's is still in use. Methods of auxiliary physical measurements, such as those of dose distribution round these units, which are now routine, also claimed his attention.

He collaborated with Dr. F. G. Spear (1932) in an early quantitative study of the influence of gamma-ray intensity on the inhibition of cell division in tissue cultures (Spear and Grimmett 1933). This work caught the eye of Professor McLennan, Chairman of the Executive Committee of the Radium Beam Therapy Research, who persuaded him to join that body. He served it, and the Radiotherapeutic Research Unit of the Medical Research Council which assimilated it, until 1944.

Grimmett excelled in the design of apparatus; he was an artist in temperament. The common view that if a scientific apparatus works it is satisfactory was foreign to him. It must not only work but be well designed, elegant, and properly finished. Furthermore, each stage in its development must reach the same standard. It was not sufficient that the drawing should be adequate for the mechanic's use; the penmanship must be excellent also. He saw everything in a big way. The idea of improving the collimation of a radium beam by fitting a solid gold nozzle to the teleradium unit was typical, and he made it a reality (Flint, Grimmett, Rock Carling and Cade 1934). He spent time and money without inhibition or fear to get the results he wanted. He supplied most of the ingenious ideas for the 5-gramme radium unit (later 10 grammes), with pneumatic transference of the radium (Grimmett 1937), replicas of which have been installed in hospitals all over the world. He was one of the first to realize the importance of the total absorption of radiation by a patient (now known as 'volume dose'), as distinct from the absorption in the malignant growth itself. He tackled its measurement practically by the construction of a full-size human shape of parallel celluloid sheets with narrow air gaps which formed ionization chambers (Grimmett 1942).

He believed in the need for higher voltages in x-ray therapy, and in 1939 began the design of what ultimately became the first high pressure Van de Graaff generator to operate in Britain. Harassed by the war, the work was moved from the bombed Radium Institute to the Imperial College of Science and then to the Hammersmith Hospital, advancing slowly, since each stage must reach perfection without short cuts. Concurrently he supervised the physical side of a long clinical experiment which compared the relative merits of radium beam and 200 kv. x-ray treatments, with the fields of both types arranged to have closely matched dose distributions. At Hammersmith he built up a fine team and a magnificently equipped physical laboratory.

Then in 1944 he disappeared from his usual scenes, reappearing briefly from time to time from Paris, or wherever else his service for the British Council, UNESCO, and UNO had taken him. Finally it was reported that he had settled in Houston, Texas, where he was developing a radiobiological research laboratory in the really big way which suited his temperament.

Grimmett's career might suggest the dominant, aggressive personality, yet he was kind, gentle, always soft-spoken, and quite imperturbable except in sympathy for others. As we walked down Regent Street in 1934 we passed seedy, out-at-elbow violinists playing in the gutter. He was filled with distress—"I know this kind of man; I have played with them. They are good musicians displaced by the 'Talkies'". He declined to assist in the atom bomb development. "The M.R.C. were approached", he said, "but I asked them to squash the idea"; then, with a wry grin in oblique explanation, "I don't mind killing Germans in odd ones".

Grimmett showed that, with few initial advantages other than talent, courage, and tenacity, he could be a '20th century' adventurer in the best sense, and at the same time make substantial contributions to human welfare. All who knew him will miss him greatly, and feel deep sympathy with his widow.

JOHN READ.

Flint, H. T., Grimmett, L. G., Rock Carling, F., and Cade, S., 1934, *Brit. Med. J.*, 14 April.
Grimmett, L. G., 1937, *Brit. J. Radiol.*, **10**, 105; 1942, *Ibid.*, **15**, 144.
Spear, F. G., and Grimmett, L. G., 1933, *Brit. J. Radiol.*, **6**, 387.

JOHN R. LOOFBOUROW

Dr. Loofbourow, who had been a Fellow of the Physical Society since 1941, died in Boston, Massachusetts, on 22nd January 1951. At the time of his death he was a professor of biophysics at the Massachusetts Institute of Technology, where he was in addition Executive Officer of the Biology Department and Chairman of the Faculty.

Loofbourow, the son of John Wilson and Henrietta Loofbourow, was born in Cincinnati, Ohio, on 1st November 1902. He was educated at the University of Cincinnati, where he obtained the degree of Bachelor of Arts in 1923. He served as an instructor in Physics there from 1925 to 1929, and as a research associate for the following six years. In 1936 the University of Dayton in Ohio awarded him the honorary degree of Doctor of Science, and he became professor of Biology at that institution. From 1935 to 1940 he was a research professor at the Institution Divi Thomae in Cincinnati, and in 1940 he was called to the Massachusetts Institute of Technology, where he was made an associate professor of biophysics.

During the war years Dr. Loofbourow was Executive Secretary of the Radar Division of the National Research Committee. He served as a special adviser to the Atomic Energy Commission in 1947 and 1948, in which year he was awarded the Presidential Certificate of Merit.

Dr. Loofbourow was a Fellow of the American Association for the Advancement of Science, the American Physical Society, the American Academy of Sciences, the New York Academy of Sciences, the Optical Society of America, the Beta Theta Pi. He was also a member of the Chemical Society, the Faraday Society and the Biochemical Society of England, as well as the Physical Society.

Dr. Loofbourow's untimely death from a heart complication was tragically followed by that of his wife, a practising physician, from the same cause within ten days. They are survived by a son, John W. Loofbourow.

Professor Loofbourow was a frequent contributor to the literature of biophysics, specializing in the field of absorption spectroscopy, particularly in the ultra-violet region. At the time of his death he was engaged in important work on the purines and pyrimidines, and was engaged in developing new techniques for absorption studies at extremely low temperatures. With Professor Richard C. Lord and the undersigned, he was the author of the text *Practical Spectroscopy* published in 1948.

With his unassuming and friendly manner, breadth of interest, and complete grasp of his subject, Professor Loofbourow had won a host of friends among both colleagues and students. As a member of a committee on improving educational methods and as Chairman of the Faculty he had contributed much to the academic community. His tolerance and quiet competence left a deep impression on all who came in contact with him, and contributed greatly to the present feeling of loss among his colleagues, students, and many friends.

GEORGE R. HARRISON.

EDWARD MALLETT

Edward Mallett, born in 1888, was educated at Southend High School and University College, London. He served an apprenticeship with Messrs. Crompton & Co., Chelmsford, and entered the Post Office Engineering Department through a Civil Service competition in 1909. Following a short period of training, he was posted to the West Section of the Scotland District and remained there until the outbreak of war in 1914. He was interested in the Territorial Army and as a result, as soon as war was declared, was posted to the 52nd Lowland Divisional Signals and saw service in the Middle East area of hostilities, finally leaving the Service with the rank of Captain.

After the war, Mallett was posted to the Transmission Section of the Engineer-in-Chief's Office and here his grasp of electrical fundamentals combined with his abounding energy found outlet in dealing with the problems of telephone transmission, which was then entering into a new phase due to the introduction of the thermionic valve. There is no doubt that Mallett, had he remained in the Post Office service, would have had a brilliant future, but a friendship made during the war years turned his energies in another direction and he left the Post Office service in 1920. It was during this period of service in the Post Office that he carried out his researches into the vibrational modes of a circular diaphragm which led to the work for which he was eventually granted his doctorate.

In September 1921 he joined the academic staff of the City and Guilds College, South Kensington, as Assistant Professor of Electrical Engineering, which post he held until September 1932.

During these eleven years he was developing and expanding the 'light current' engineering section of the department. A considerable expansion was taking place under the direction of Professor Fortescue, and Mallett, with his active and fertile mind, backed by his considerable experience in the Post Office Engineering Department, found scope for developing the instruction and research in electrical communication.

He had an intuitive engineering sense and considerable academic skill, and these combined produced a lecturer of inimitable style and a research worker capable of producing results from the minimum of apparatus. He was extremely popular with both staff and students and joined in many of their social activities.

The title of Reader was conferred on him in 1927 by the University of London, and from that date until his death he remained a member of the Board of Studies in Electrical Engineering. He was for several years Secretary to the Board of Studies, acquiring an intimate knowledge of the activities of the Board and of the University, which knowledge served them well over the period of his long membership.

In addition to these activities he still found time to write several papers on a variety of subjects of interest to the communications engineer which he presented to the Institution of Electrical Engineers and to the Physical Society.

In July 1926 he was awarded the Degree of D.Sc.(Eng.) as an Internal Student of University College, London, the thesis title being *Forced Oscillations, Electrical and Mechanical*. He was frequently consulted by outside bodies and the University on electrical engineering matters, and in 1929 he produced his textbook on Telegraphy and Telephony which became, and still is, a valuable foundation for students of electrical communication.

It was at the height of his academic activities that he left the City and Guilds College to become Principal of Woolwich Polytechnic, but although he became more and more immersed in administrative studies, he never lost contact with the College or the University of London.

As Principal of Woolwich Polytechnic he was intensely interested in the University aspect of the Polytechnic but at the same time took a very active interest in the large non-university side of Polytechnic activities. He was a keen believer in the National Certificate Course for part-time and evening students, and had a place in his heart for the craftsman, ensuring that he was provided with proper shops and tools. It was during his Principalship that the Sandwich Course in Engineering was started, and this eventually developed into the Engineering Degree Course of today. This was soon followed by Degree Courses for B.Sc. (Chemistry, Physics, Mathematics), and the full-time School of Art Course.

The Polytechnic Union came into being during his time. He saw that the union of the diversified clubs and societies of the Polytechnic into one corporate whole could be of great value to the students and members, and the passing of time has proved him correct. The success of the Union in all its aspects is a tribute to him: he gave it a free, unfettered hand, and held it in high esteem.

Extensions to the Polytechnic were made during his time, and future developments were being planned by him when his fatal illness intervened.

He was a member of the Institution of Civil Engineers, the Institution of Electrical Engineers, the Institute of Physics and the Physical Society (since 1921), and a Fellow of University College.

Edward Mallett, D.Sc.Eng. (Lond.), M.Inst.C.E.. M.I.E.E., F.Inst.P., Fellow of University College, London, died on the 10th December 1950. C. R. URWIN.

HANTARO NAGAOKA

PROFESSOR HANTARO NAGAOKA, who had for nearly forty years occupied a worthy place among the few Honorary Fellows of the Physical Society, died on 10th December 1950 at the age of 85. In the English-speaking world his name is probably most familiar in connection with ' Nagaoka's formula ' for the calculation of inductance, and indeed his work on the subject has been of great value to workers in the radio field and to physicists concerned with precision measurements in inductance. For example, in addition to his formulae he published, with S. Sakurai, tables which give, with very little computation, values for the self-inductance of single layer coils, and for the mutual inductance of coaxial circles to six-figure accuracy. This work, however, is hardly representative of his main interest; he was essentially an all-round physicist in the Rayleigh, Kelvin, J. J. Thomson tradition, and one who could refer to physics as 'the Study of Nature'.

He was born in 1865 and was of honourable lineage. He graduated in physics at the Tokyo Imperial University in 1887, a time when, as he recalled later, the library in the physics department occupied but a single shelf six feet wide, and the only periodical available was the *Philosophical Magazine*. Nevertheless he must have immediately embarked upon experimental research in magnetism, for papers published by him in 1888, 1889 and 1890 are discussed in some detail by Ewing in his *Magnetic Induction in Iron and Other Metals*. Among other things Nagaoka showed that in certain circumstances the magnetization of nickel is reversed by torsion, an effect so surprising that both Kelvin (then W. Thomson) and Weiss had it confirmed in their own laboratories. Sixty years later Nagaoka remarked that the reason why still remained quite unknown. He continued his work in Tokyo for six years, took his doctorate in 1893, and then proceeded to Germany, where he studied for three years, attending lectures by Helmholtz, Boltzmann, Planck and others, and working on magnetostriction. During this time he visited the Physikalisch-Technische Reichsanstalt and was received by Lindeck who, having recently translated Ewing's book into German, knew of his work. Lindeck asked him when he had left Ewing's laboratory at Cambridge, and when he learnt that the work had been done several years earlier in Japan was sufficiently impressed to show him the laboratory in detail. Nagaoka with a pleasant touch of humour said later that he realized then that a bit of good work is a much better introduction than a distinguished visiting card.

Returning to Japan, he was appointed Professor of Physics at his old university, and for the next fifty years and more he flourished as an active and versatile leader of physics in Japan, occupying positions of increasing responsibility and influence, which in view of his own sentiment mentioned above need not be set out in detail here.

Much of his early work was done on geophysics with Professor A. Tanakadate who was at that time Director of the Physical Institute of the university; a little later he returned to magnetism and with Professor K. Honda investigated the magnetostrictive properties of nickel-steels of various nickel contents; they showed that changes of volume, as well as of length, occur. The geophysical work included the determination of ***g*** at Tokyo, comparisons of ***g*** at Tokyo and Potsdam, and a systematic study of the elasticity of rocks in relation to the propagation of seismic waves through the earth's crust. His interest in geophysics remained with him until the end of his life, but round about 1900 he seems to have been strongly attracted by two widely different scientific developments : atomic physics arising from the discoveries of the electron, x-rays and radium on the one hand, and wireless communication on the other. His interest in the atom ultimately gave rise to a very considerable body of detailed work on spectroscopy, hyperfine structure and the Zeeman effect, carried out with Dr. T. Takamine. His interest in ' wireless ' seems to have led to his work on formulae for inductance calculation, already mentioned, and to a consideration of the effect of the ionosphere, familiar to him from its association with terrestrial magnetism which he had investigated earlier, on radio communication. He foresaw that the solar eclipse of 1914 would provide an opportunity of testing the Kennelly–Heaviside theory, if the ionized layer arises from the photoelectric effect of solar radiation. He made a calculation of the effect that should be observable on suitable wireless signals. Unfortunately the first world war prevented the making of the observations, but the detailed observations made in the U.S.A. at the eclipse of 1925 showed that the idea was sound although, in the light of later information, Nagaoka's original calculation was of course only the first rough approximation.

It is estimated that Nagaoka published in all some 300 papers, and it will be realized from what has been said that they are so diverse in character that they do not lend themselves to brief effective description. What does stand out very clearly is his life-long devotion to physical science : we see him as mathematical physicist, an experimenter—he devised a magnetograph, a silica pendulum for gravity determinations and spectroscopic devices—a patient observer—he himself is said to have made fifty determinations in the gravity survey that he organized—and an original thinker. On this last point it must be noted that he had the honour of being the first to suggest a planetary structure for the atom. Rutherford in *Radioactive Substances and their Radiations* wrote " This type of ' Saturnian ' atom was first suggested in 1904 by Nagaoka, who investigated mathematically the stability of an atom composed of a central attracting mass surrounded by rings of rotating electrons. He showed that such a system was stable if the attractive force was large and the electrons in rapid motion ". When it is remembered that it was in this year also that J. J. Thomson was still investigating the model consisting of electrons inside a positively charged sphere, the originality of the conception can be better appreciated. It was spectroscopy rather than radioactivity that was in Nagaoka's mind; his aim was to account for the discrete spectral lines characteristic of the atom, and he thought that they might be associated with vibrations of the electrons in and perpendicular to the plane of the ring. We now know that the vital key to the problem was still missing, and that Bohr was to find it some years later, but Nagaoka's contribution commands our respect.

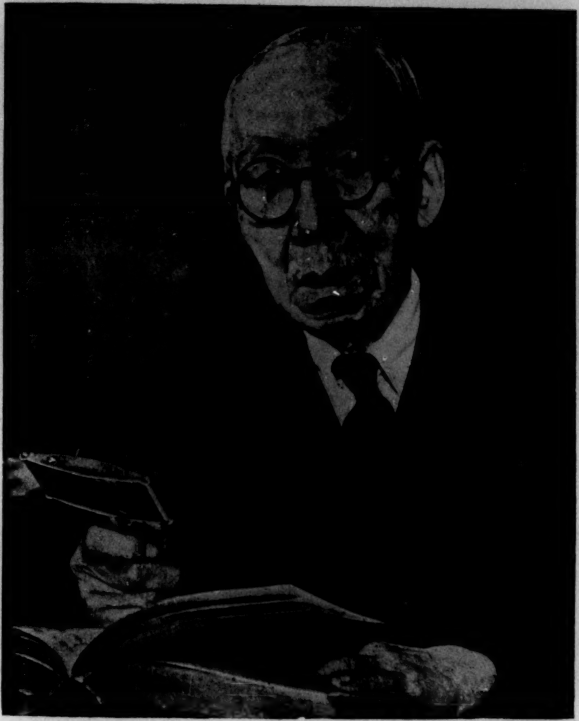
Nagaoka came to occupy an eminent position in the scientific life of Japan. He was elected a member of the Imperial Academy in 1905, became Director of the Institute of Physical and Chemical Research in Tokyo (now the Scientific and Research Institute) in 1926 and was among the first recipients of ' Bunkwa Kunsho ', a medal of great distinction, in 1937. He was President of the Imperial Academy in 1939 and 1948. He visited England on two or three occasions, was awarded an honorary degree at Cambridge, and read papers in London before the Physical Society. He was elected an Honorary Fellow of the Society in 1912. That this tribute to an outstanding fellow worker in the new school of physics then rapidly rising in Japan has survived the shock of two world wars may serve as a welcome reminder that physics still stands as a co-operative human enterprise.

L. H.

FRANK BENNETT YOUNG

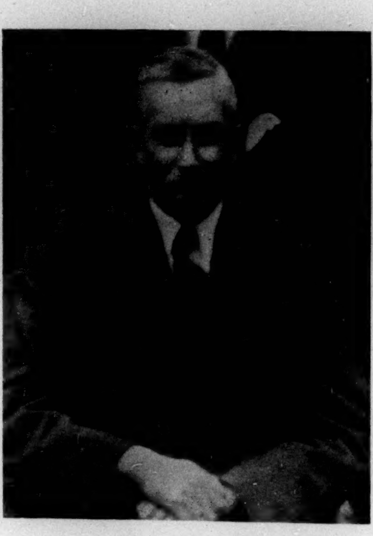
It is recorded with regret that Frank Bennett Young died on 29th January 1951. He was a Fellow of the Physical Society from 1916 to the time of his death, and a founder Fellow (1920) and life member of the Institute of Physics.

He was born in 1877 in Bristol where he received his early education. In 1896 he went to the Borough Road College, Isleworth, where he was awarded a scholarship enabling him to spend a year at Jena University. In 1899 he obtained the B.A. degree of London University. At this time he became interested in science from a philosophical point of view and began to study physics, returning to Bristol to teach physics in a Higher Grade School. During this period he attended evening classes at University College, Bristol, before the foundation of the University. It was there he met Professor Chattock who had an important influence on his career. His enthusiasm for research in physics increased and he spent all his spare time (evenings, week-ends and holidays) on research under Chattock's guidance.



Photograph by Shigeru Tamura

HANTARO NAGAOKA
(1865-1950)



DR. F. B. YOUNG, O.B.E.

(*enlarged from a small group photograph*)

His published papers at this time included "The field at the point of an electrified needle" (*Phil. Mag.*, 1907, **13,** 542) and "Critical phenomena of ether" (a study of the opalescent phenomena near the critical point) (*Phil. Mag.*, 1910, **20,** 793). On the foundation of the University of Bristol he took advantage of one of its early ordinances which permitted him in 1910 to present these publications for the degree of B.Sc. by research. In 1915, soon after the outbreak of the 1914–18 war (Prof.) A. M. Tyndall obtained for him the job of localizing shrapnel bullets in the body by x-rays at the Bristol Royal Infirmary, a problem quite beyond the medical man of that day. This was the first time a physicist had entered its doors to give service.

In 1916 he joined the Admiralty, for anti-submarine research under the Board of Invention and Research (W. H. Bragg and Rutherford were on the controlling committee). He worked in Admiralty research establishments at Aberdour (Fife), Parkeston Quay (Harwich), Shandon (Gareloch) and, after the war, at Teddington (Middlesex). In the earlier days he was mainly concerned in research work on hydrophones and in underwater acoustics. Some of this work was published after the war, viz. "On light body hydrophones and the directional properties of microphones" (*Proc. Roy. Soc.* A, 1921, **100,** 252) and "On the acoustic disturbance produced by small bodies on plane waves transmitted through water, with special reference to the single-plate direction finder" (*Proc. Roy. Soc.* A, 1921, **100,** 261). This had an important bearing on the design of directional hydrophones. At a later stage of the 1914–18 war he carried out important researches on the generation of electric currents in the sea (*a*) due to the presence of steel ships, and (*b*) due to the tidal flow of a conducting medium (the sea) in the earth's field. Some of the experiments and conclusions reached in the latter were published in *Phil. Mag.*, 1920, **40,** 149, in an article entitled "Electrical disturbances due to tides and waves" by F. B. Young, H. Gerrard and W. Jevons. At this time he was also interested in the development of a new method of locating faults in submarine cables—using alternating currents and silver-chloride electrode search gear. This method has since been extensively used by cable companies and by the G.P.O. Soon after the end of the 1914–18 war he obtained the degree of D.Sc. (Bristol 1919). On the removal of the Admiralty research establishment from Shandon (Gareloch) to Teddington he worked on course plotters for ships and developed what is now known as the A.R.L. (Admiralty Research Laboratory) Course Plotter. Another line of research which he directed related to the preservation of the rubber insulation of cables in ships. It will be understood that much of his research work for the Admiralty remained unpublished on grounds of security. In July 1934 he became Superintendent of the Admiralty Research Laboratory, Teddington, a post which he filled until his retirement on reaching the age limit in October 1937. Whilst in this post he was awarded the O.B.E.

In 1939 he returned to Admiralty work in Portsmouth soon after the outbreak of the 1939–45 war, taking charge of an important section of research on the degaussing of ships as a protection against enemy magnetic mines. He finally retired from Admiralty service in 1941, taking on various unpaid wartime duties in Bristol.

He eventually settled in Portsmouth after the war where he revived his old-time interests in music and in sketching and water colour painting, He was of a retiring disposition, very reliable, thorough and conscientious in all his work, and generous in giving more than their share of credit to others who worked with him. He will be missed by those who, like myself, knew him well.

A. B. WOOD.

ARNOLD SOMMERFELD

On 26th April 1951, Arnold Sommerfeld, Honorary Fellow of the Physical Society, died from the consequences of a street accident at the age of eighty-two.

He was born in 1868 at Konigsberg, East Prussia, and attended school and university in his native town. In 1893 he went to Gottingen and came under the influence of Felix Klein. Here he had a thorough mathematical training and acquired the skill in the theory of differential equations, analytical functions and complex integration which he later applied with great success to physical problems. The direct fruit of his co-operation with Klein was their famous book on the spinning top (1895–1910).

In 1895 Sommerfeld became lecturer in Göttingen, in 1897 professor at the Mining College in Clausthal and in 1900 he was appointed to a chair of mechanics at the Technical

College in Aachen. From 1906 until his death he was professor of theoretical physics in Munich. He declined several attractive positions, as for instance the chair of Planck in Berlin.

Sommerfeld's scientific work covers a vast field of which only some major points can here be mentioned. His contributions to classical physics were in hydrodynamics, by work on the stability of laminar motion, and in optics, which he enriched by ingenious rigorous solutions of some diffraction problems. But soon he directed his attention to the new ideas of relativity and quantum mechanics. Combining these two subjects he developed Niels Bohr's theory of atomic spectra by taking into account the relativistic variability of mass, and obtained in 1915 his celebrated fine structure formula for hydrogen-like spectra. Sommerfeld's fine structure constant dominates not only these phenomena but the whole of quantum electrodynamics. In 1917 be hegan to write his great book on the structure of atoms and spectral lines which has so immensely contributed to the understanding and general acceptance of quantum theory. The advent of quantum mechanics led him first to write a short complementary volume which was later extended to a comprehensive representation containing a wealth of information. The study of wave mechanics in combination with Pauli's principle led Sommerfeld to another great success, its application to the electron theory of metals.

Sommerfeld was a brilliant teacher and attracted pupils from all over the world. When in 1938 he retired from his Chair, he began to prepare the publication of his lectures, of which all but one volume have appeared, and it is hoped that this last one will also be published. His style and power of representation is such that his books read like novels. But still greater was the charm of his personality which won him the devotion of his pupils, amongst whom were Pauli, Heisenberg, Brillouin, F. London and many other outstanding men.

Sommerfeld was a member of many academies and learned societies, amongst them the Royal Society. He received the Max Planck Medal of the German Physical Society and many other honours. He was at one time Karl Schurz Professor at the University of Wisconsin and visiting professor at the Californian Institute of Technology and at several other American universities.

MAX BORN.

STUART ARTHUR FRANK WHITE

STUART ARTHUR FRANK WHITE, who died on 12th January 1951, aged 80 years, was elected a Fellow of the Physical Society in 1895. He was for many years Professor of Mathematics at King's College, London, Dean of the Faculty of Science and latterly Vice-Principal and Fellow of the College. He retired in 1933 after occupying his Chair for thirty years.

White's education began at Portsmouth Grammar School from which he proceeded (with a scholarship or exhibition, I believe) to Wadham College, Oxford, at Michaelmas 1887. In his student days he seems to have been as much interested in physics and astronomy as in mathematics, and before going to King's College he held an appointment at Oxford under Professor Clifton. In 1893 he was appointed Demonstrator of Natural Philosophy—strangely enough, on the recommendation of the Engineering Committee—under Professor Grylls Adams, and in 1903 he succeeded W. H. Hudson in the Chair of mathematics. The appointment to this Chair of the demonstrator of natural philosophy, rather than of a mathematician *vom Fach*, appears to have been due to the desire of the Council to infuse a more vivid and practical spirit into the teaching of mathematics to the students of such subjects as engineering, physics, etc., whose interest lay less in mathematics for its own sake than in its practical applications. For such students White was ideal. He was a most able teacher, lucid and always interesting. He was moreover whole-heartedly devoted to his teaching duties and to the progress and welfare of his students, whose affection and esteem he undoubtedly won.

White was a keen golfer and skier and for a long period spent part of every Christmas vacation in Switzerland. He was intensely devoted to the college he served and his wise counsel and guidance were greatly valued, not only by the Professorial Board of the College, but also by his mathematical colleagues in the university.

WM. WILSON.

Miss JESSIE S. CHREE

Fellows of the Physical Society will be sorry to learn of the death of Miss Jessie S. Chree, sister of the late Dr. Charles Chree, F.R.S.

Dr. Chree was at one time President of the Physical Society, and the Society's *Charles Chree Medal and Prize* was instituted in memory of him through the generosity of Miss Jessie Chree.

ALLAN FERGUSON

We record with regret the death on 9th November 1951 of Dr. Allan Ferguson who was the Papers Secretary of the Physical Society from 1928 to 1938 and President from 1938 to 1941.

An appreciation will be published with next year's Obituary Notices.

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ABSTRACTS FOR SECTION A

The Temperature Effect of Extensive Air Showers, by A. L. Hodson.

ABSTRACT. Extensive air showers have been recorded at sea level over a period of nine months. The data show a fairly good correlation between the shower rate and the air temperature at about one cascade unit above the apparatus, e.g. at the 950 mb. level. A temperature coefficient of $-(0{\cdot}38\pm0{\cdot}11)\%$ per °C. at this level is indicated. This may be explained by the dependence of the lateral spread of an extensive shower on the air density.

The Disintegration of Light Nuclei by Meson Capture, by A. C. Clark and S. N. Ruddlesden.

ABSTRACT. The disintegration by π^--meson capture of the elements helium and carbon is discussed. Two types of meson theory, scalar and pseudo-scalar, are considered. The relative probabilities of the various modes of disintegration of helium are calculated and the energy spectra of the most likely one, viz. proton emission, are shown. The disintegration of carbon into two α-particles and one singly charged particle is considered, using a simple α-particle model. The energy spectra and angular correlations of the α-particles obtained by a direct transition and by a transition via an excited state of ^{8}Be are compared.

Range–Energy Relation for Protons in Ilford C2 Dry Emulsion, by F. A. El-Bedewi.

ABSTRACT. The range–energy relationship for protons of energies up to 12 MeV. in Ilford C2 dry emulsion has been determined with an accuracy better than $0{\cdot}5\%$ in energy. The particles were produced by bombarding a thin beryllium foil with deuterons having an energy of 8 MeV. The energy of the incident deuterons was obtained by a method which involves the measurement of the two angles of emission at which the two proton groups of longest range from the reaction ^{9}Be(d, p)^{10}Be have the same energy.

The Differential Cross Section for High Energy Nucleon–Nucleon Collisions and the Mean Square Angle of Scatter, by H. S. Green and H. Messel.

ABSTRACT. In an attempt to account theoretically for the lateral and angular development of the penetrating particles in high energy cosmic-ray showers, we have developed a method for deriving the differential from the total cross section, in the laboratory frame of reference. An integral equation is set up for the differential cross section, and solved assuming that the differential cross section in the centre-of-mass frame of reference is of the form $R+S'\cos^2\theta$, where R and S' are functions of the incident and scattered energies only. Using a form of the total cross section derived from the analysis of high-energy cosmic-ray experiments, the mean square angle of scatter in a nucleon–nucleon collision has been calculated and found to be inversely proportional to the energy of the incident particle. The results are compared with experiments.

Nuclear Spin and Magnetic Moment of Rhodium $^{103}_{45}Rh$, by H. Kuhn and G. K. Woodgate.

ABSTRACT. The radiation from a hollow-cathode tube containing rhodium was passed four times through an atomic beam of rhodium. Absorption was found in eight lines, four of which showed hyperfine structure doublets of separation $0{\cdot}023$ cm^{-1}. The splitting could be ascribed to the ground state $4d^8\,5s\,^4F_{9/2}$. It was concluded with certainty that the spin of the rhodium nucleus $^{103}_{45}$Rh has the value $I=\frac{1}{2}$. The relative intensities of the components show that the magnetic moment is negative. The observed splitting leads to an estimated value of $\mu=-0{\cdot}10$ nuclear magneton. The values of I and μ indicate a configuration $(5g_{9/2})^6 3p_{1/2}$ for the last seven protons of the rhodium nucleus in the Goeppert-Mayer model.

Slow Neutron Scattering by Ferromagnetic Crystals, by R. G. Moorhouse.

ABSTRACT. Consideration is made of the predictions of the Bloch spin wave model within its region of approximate validity, for the magnetic scattering of very slow neutrons by a ferromagnetic. It is found that observable peaks of intensity in the angular distribution of scattered neutrons occur due to spin wave absorption. Neutrons are also scattered with phonon absorption and by an analogous calculation it can be shown that similar peaks occur due to this vibrational scattering. A possible method is suggested of differentiating between the two kinds of scattering in order to provide a check of the Bloch model.

Antiferromagnetism by the Bethe Method, by J. M. Ziman.

ABSTRACT. The Bethe method for order–disorder is applied to a lattice of magnetic dipoles with antiferromagnetic nearest-neighbour interactions. Magnetization and entropy are calculated for all values of temperature and magnetic field, and an anti-parallel ordered phase is found below a definite transition curve. The computed curves agree qualitatively with those observed experimentally with certain paramagnetic salts at low temperatures

Asymmetrical Electron Diffraction Effects from Single Crystals of Silver, by D. W. Pashley.

ABSTRACT. Silver crystal surfaces which had been prepared by either electrolytic etching or electrolytic polishing were studied by means of the electron diffraction reflection technique. It was found that fine structure effects, in the form of split spots or streaks through spots, gave information concerning the topographies of the various surfaces of the crystals. The fine structure effects showed certain asymmetries which do not immediately follow from the Laue diffraction theory. The possible causes of these asymmetries are discussed, and it is concluded that they were probably due to distortion of the silver crystals, but that some might have also been due to refraction effects.

Field Dependence of the Dielectric Constant, by J. J. O'Dwyer.

ABSTRACT. A very general calculation has been made by Fröhlich giving the dielectric constant of a material in terms of the spontaneous fluctuations of the electric moment of the material which would occur in a field-free region. This calculation is now extended one step further so as to include the first term in the field dependence of the dielectric constant. By way of illustration the general theory is applied to Kirkwood's model for a dipolar liquid. (In order to preserve continuity of argument it has been found necessary to reproduce much of Fröhlich's work.)

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